A Level Maths Predicted Papers 2025 (June)

Paper 1: Pure Mathematics 1: Set 1

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Telephone No: **07828343435**

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This AL Maths paper 1 (Predicted Paper 2025: Set 1) has been created based on the most common topics from previous past papers. This paper should be excellent for helping students revise for exams; however, it should not be relied upon as the sole basis for revision.

Grade Boundary: A: 60% and A*: 80%

1.

Given that θ is measured in radians, use small angle approximations to simplify the following expression.

$$\frac{\cos 6\theta - 1}{6\theta \sin\theta}$$

(3)

2.

The curve defined by the equation $y = 3 \times 4^x$ intersects with the curve defined by $y = 25 - 4 \times 4^{x+2}$ at the point K.

Using algebraic methods, determine the exact x-coordinate of the point K

(3)

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3.

(a) Find $\overrightarrow{AB}$

where

Point A has position vector 3i+4j-5k

Point B has position vector 5i+7j-3k

Point C has position vector 8i+12j+8k

Wehere O is Origin

(b) Find the angle of ABC

(c) Show that quadrilateral OABC is trapezium

(4)

4.

Let $y = k^x$ where $k > 0$ and $k \neq 1$.

a) Prove that $\frac{dy}{dx} = k^x \text{ In } k$.

A curve has the equation $y = \sin(x) + 4 \times 2^{-x}$, where $x > 0$.

b) Find the equation of tangents at the point of the curve where $x = \pi/3$

(5)

5. A curve has parametric equations

$$x = 7 + 2sint \; t \in \mathbb{R}$$

$$\frac{y}{2} - cost = 1.5$$

(a) Show that a Cartesian equation of the curve C is $(x - a)^2 + (y - b)^2 = r^2$ and hence sketch the curve.

The line *l* has equation *y* = 7*x* + *p* where *k* is a constant.

Given that *l* is a tangent to *C*,

(*b*) find the possible values of *p*, giving your answers as simplified surds. **(5)**

6. (a) Express $2 \sin x + 5 \cos x$ in the form $R \sin (x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

The temperature, θ °C , inside a room on a given day is modelled by the equation

$$\theta = 7 + 2\sin\left(\frac{\pi t}{16} - 5\right) + 5\cos\left(\frac{\pi t}{16} - 5\right) \qquad 0 \leq t < 32$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(6)

7.

$$g(x) = 3x^2 + 6x + 15 \quad x \in \mathbb{R}$$

(*a*) Write g (x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(*b*) Sketch the curve with equation $y = \mathrm{f}(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(*c*) (i) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where

$$g(x) = 3(x - 2)^2 + 6x - 22 \qquad x \in \mathbb{R}$$

(ii)Find the range of the function $0 < \theta < 2\pi$

$$\frac{25}{3\sin^2\theta + 6\sin\theta + 15}$$

$$x \in \mathbb{R}$$

(8)

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8.

(a) Find the first four terms, in ascending powers of *x*, of the binomial expansion of

$$\frac{2x+3}{\sqrt{25-16x}}$$

writing each term in simplest form.

(b) State the range of values of x for which each expansion is valid

(4)

9.

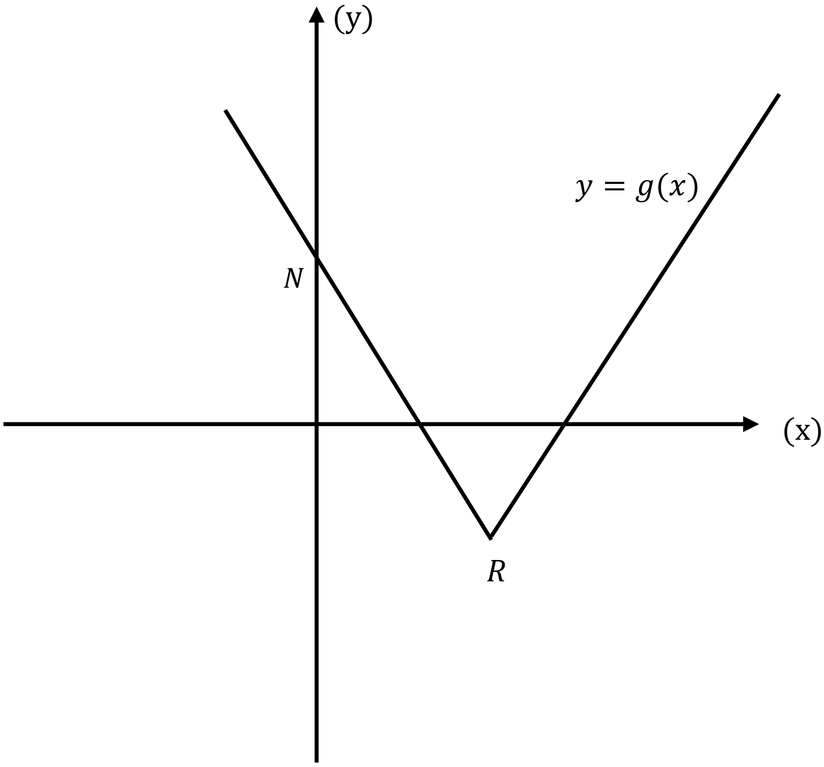

Figure shows the equations of $y = g(x)$, where

$$g(x) = 5|px - 4| - 3$$

and p is positive constatnt

The graph $y = g(x)$ intersects the Y-axis at the point N and has a mimium point at R

(a) Find, the x coordinate of R in terms of p

(b) Final the y coordinate of N

Given that the $y = 3 - 4x$ intersects the graph y=g(x) at (i) at two distint points (ii) at one point (iii) does not intersects

(c) find the range of possible values of p for all 3 cases

(d) Find the range of values of x that satisfy the inequality interms of p

$$5|px - 4| - 3 < 2x + 5$$

(8)

10.

(*a*) Evaluate:

$$\int x(x^2+1)e^{x^2+1}\,dx$$

(*b*) Evaluate:

$$\int (3\theta + 4\cos 3\,\theta)^2\,d\theta$$

(7)

11.

A Student is studying the number of rabbits and the number of cats on village.

The number of rabbits, measured in thousands, R, is modelled by the equation

$$R = 75 + 25\,e^{0.07t}$$

where t is the number of years from the start of the study.

According to the model,

(a) find the number of rabbits at the start of the study,

The number of cats, measured in thousands, C, is modelled by the equation

$$C = 30 + 40e^{0.14t}$$

where t is the number of years from the start of the study.

When $t = t_1$, according to the models, there are an equal number of rabbits and cats.

(b) Find the value of t_1

(c) Hence find the rate of increase in the number of rabbits in this population 15 years from the start of the study

(8)

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12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove the trigonometric identity:

$$sin3A = 3sinA - 4sin^3A$$

(b) Using the result from part (a), solve the equation: $-180 \leq x \leq 360$

$$1 - sin3x = cos^2x$$

(6)

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13.

(a) The first three terms of an arithmetic sequence are:

$$\ln(2), \qquad \ln(2^k + 1), \qquad \ln(2^{k+1} + 2).$$

Find the exact value of k.

(b)

Find the value of this series

$$\sum_{n=3}^{\infty} \cos(180n)^0 \left[\frac{3}{4}\right]^n$$

(6)

14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(*a*) Using this model, show that

$$\frac{dr}{dt} = -\frac{\mu}{r^2}$$

where μ is a positive constant.

Given that

- the initial radius of the balloon is 30 cm
- after 10 seconds the radius of the balloon is 15 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(*b*) solve the differential equation to find a complete equation linking r and t.

(*c*) Find the limitation on the values of t for which the equation in part (*b*) is valid.

(8)

15. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 6 \sin 4y \qquad \frac{-p}{4} < y < \frac{p}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(c) Show that, for all points (x, y) lying on C,

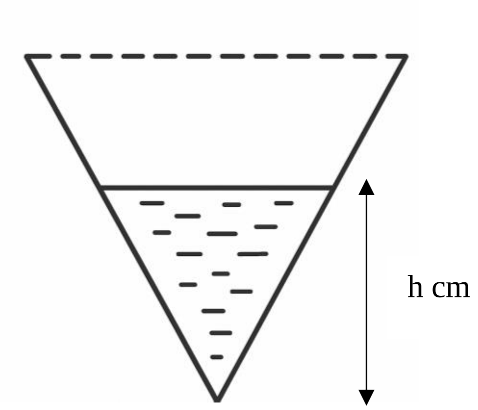
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(5)

16.

The diagram shows a container shaped like a right circular cone. Water is poured into the container, but it slowly leaks out through a small hole at the tip.



It is assumed that the volume of water, V cm³, decreases at a rate proportional to its current volume. The depth of the water at time t minutes is h cm.

(a) Show that $\frac{dh}{dt} = -\lambda h$ where λ is a positive constant.

Given that:

- h=15 when t=0
- h=5 when t=25

(b)

(i) Show that $h = Ae^{kt}$

(ii) Find the exact value of A and k.

(c)

Find the time t when the depth of water has reduced to 3 cm.

(10)

TOTAL FOR PAPER IS 100 MARKS

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Paper 1: Pure Mathematics 1: Set 1: Answers

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1)

$$\frac{\cos 6\theta - 1}{6\theta \sin\theta}$$

$$= \frac{\frac{1-(6\theta)^2-1}{2}}{6\theta \times \theta}$$

$$= \frac{-(36\theta^2)}{12\theta^2}$$

$$= (-3)$$

2) $y = 3 \times 4^x$

$y = 25 - 4 \times 4^{x+2}$

$$3 \times 4^x = 25 - 4 \times 4^x \times 16$$

$$67 \times 4^x = 25$$

$$4^x = \frac{25}{67}$$

$$x = ln\left(\frac{25}{67}\right) \times \frac{1}{ln(4)}$$

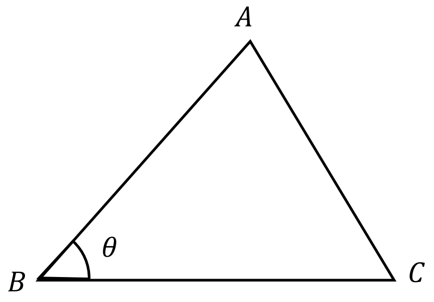
$$x = \frac{ln\left(\frac{25}{67}\right)}{ln(4)}$$

3) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

a) $= \left(5\underset{\sim}{i} + 7\underset{\sim}{j} - 3\underset{\sim}{k}\right) - \left(3\underset{\sim}{i} + 4\underset{\sim}{j} - 5\underset{\sim}{k}\right)$

$= 2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$

b)



$|\overrightarrow{AB}| = \sqrt{4+9+4} = \sqrt{17}$

$\overrightarrow{BC} = \begin{pmatrix} 8 \\ 12 \\ 8 \end{pmatrix} - \begin{pmatrix} 5 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 11 \end{pmatrix}$

$|\overrightarrow{BC}| = \sqrt{9+25+121} = \sqrt{155}$

$\overrightarrow{AC} = \begin{pmatrix} 8 \\ 12 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 13 \end{pmatrix}$

$|\overrightarrow{AC}| = \sqrt{25+64+163} = \sqrt{258}$

$\cos\theta = \frac{17+155-258}{2(\sqrt{17})(\sqrt{155})} \Rightarrow \theta = 146.9°$

$\theta = 147°$

c) $\overrightarrow{AB} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

$\overrightarrow{OC} = 8\underset{\sim}{i} + 12\underset{\sim}{j} + 8\underset{\sim}{k}$

$= 4\left(2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

$\overrightarrow{OC} = 4\overrightarrow{AB}$

OC parell to AB

$OABC$ trapezium

4) $Y = k^x$

a) $\frac{dy}{dx} \Rightarrow \quad y = k^x$

$lny = lnk^x$

$lny = x\ lnk$

$^1/_y \times \frac{dy}{dx} = 1 \times lnk$

$$\frac{dy}{dx} = y\ lnk = k^x ln(k)$$

b) $Y = \sin(x) + 4 \times 2^{-x}$

$\frac{dy}{dx} = \cos(x) + 4(-x)ln2$

$= \cos(x) - 4x\ ln(2)$

$x = {}^{\pi}/_3 \qquad \frac{dy}{dx} = {}^1/_2 - 4({}^{\pi}/_3)ln2 \quad = {}^1/_2 - \frac{4ln2}{3} \quad , \qquad Y = {}^{\sqrt{3}}/_2 + 4 \times 2^{(-\pi/3)}$

$({}^{\pi}/_3, 2.8) \qquad\qquad = (-2.4) \qquad\qquad Y = \frac{\sqrt{3}}{2} + \frac{4}{2^{\pi/3}}$

$\Rightarrow (1.1, 2.8) \qquad\qquad Y = 2.8$

$y - 2.8 = (-2.4)(x - 1.1)$

$y = -2.4 + (-0.16)$

5) $x = 7 + 2\sin t \quad — (1)$

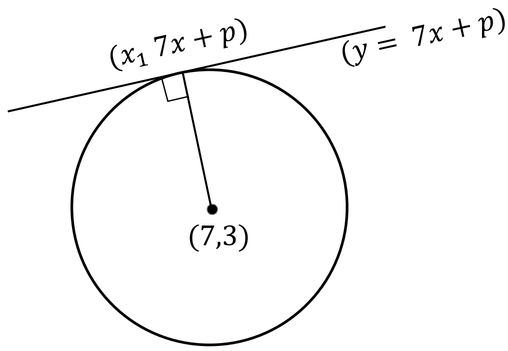
${}^{y}/_2 - \cos t = 1.5 — \ (2)$

$(1) \Rightarrow\ \sin t = \left(\frac{x-7}{2}\right) \qquad\qquad (2) \Rightarrow\ \cos t = \left(\frac{y-3}{2}\right)$

$\sin^2 t + \cos^2 t = 1$

$\frac{(x-7)^2}{4} + \frac{(y-3)^2}{4} = 1$

$(x-7)^2 + (y-3)^2 = 4 = 2^2 \qquad ; \qquad (r = 2)$



$(x-7)^2 + (7x + p - 3)^2 = 4$

$x^2 - 14x + 49 + 49x^2 + (p-3)^2 + 14(p-3)x = 4$

$50x^2 + 14(p-4)x + (p-3)^2 + 45 = 0$

$b^2 - 4ac = 0 \;\rightarrow\; 14^2(p-4)^2 - 4(50)((p-3)^2 + 45) = 0$

$14 \times 7(p-4)^2 - 100((p-3)^2 + 45) = 0$

$14 \times 7(p^2 - 8p + 16) - 100(p^2 - 6p + 54) = 0$

$-2p^2 - 184p - 3832 = 0$

$p = -60.14$ or $p = -31.86$

$(-46 - 10\sqrt{2}$ $\quad$ $(-46 + 10\sqrt{2})$

6) a)

$2\sin x + 5\cos x = R\sin x\cos\alpha + R\cos x\sin\alpha$

$2 = R\cos\alpha$ — (1)

$5 = R\sin\alpha$ — (2)

$\tan\alpha = 2.5$ $\quad$ $R = \sqrt{25+4}$

$\alpha = 1.19$ $\quad$ $R = \sqrt{29}$

b) $\theta = 7 + \sqrt{29}\sin\left(\frac{\pi t}{16} - 5 + 1.19\right)$

$\theta_{max} = 7 + \sqrt{29}$

$= 12.38°\text{C}$

c) $\quad$ $0 \leq \frac{\pi t}{16} < 2\pi$

$\frac{\pi t}{16} - 3.81 = \pi/2$ $\quad$ $-3.81 \leq \frac{\pi t}{16} - 3.81 \leq 2.47$

$t = 27.4$ hours

$(03{:}24\text{ am})$

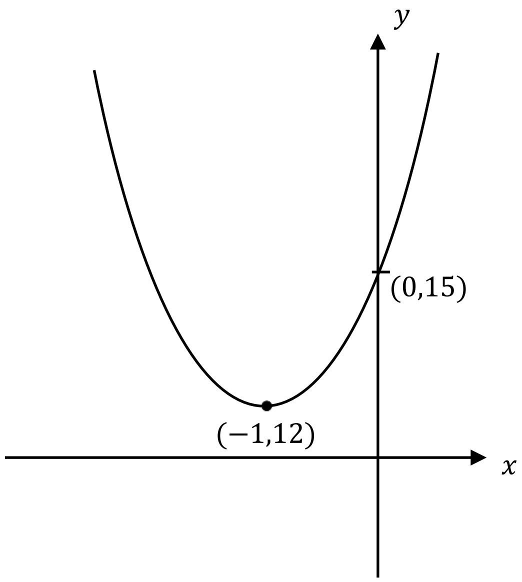
7) $g(x) = 3x^2 + 6x + 15$

a) $g(x) = 3[x^2 + 2x + 5]$

$= 3[(x+1)^2 + 4]$

$= 3(x+1)^2 + 12$

b)



c) (i) $g(x) = 3(x-2)^2 + 6x - 22$

$= 3(x-2)^2 + 6(x-2) - 10$

$= 3(x-2)^2 + 6(x-2) + 15 - 25$

$\begin{pmatrix} +2 \\ -25 \end{pmatrix}$

(ii) $0 < \theta < 2\pi$

$y = \dfrac{25}{3\sin^2\theta + 6\sin\theta + 15} = \dfrac{25}{3(\sin\theta + 1)^2 + 12}$

$y_{max} = {}^{25}/_{12}$

$y_{min} = {}^{25}/_{24} \left({}^{25}/_{24} < y < {}^{25}/_{12}\right)$

8) a)

$$\frac{2x+3}{\sqrt{25-16x}} = (2x+3)(25-16x)^{-1/2}$$

$$= \frac{2x+3}{5}\left(1-\frac{16}{25}x\right)^{-1/2}$$

$$= \left(\frac{2x+3}{5}\right)\left[1+\left(-1/2\right)\left(\frac{-16}{25}x\right)+\frac{(-1/2)(-3/2)}{2!}\left(\frac{-16}{25}x\right)^2+\frac{(-1/2)(-3/2)(-5/2)}{3!}\left(\frac{-16}{25}x\right)^3+..\right]$$

$$= \frac{3}{5}+\frac{74x}{125}+\frac{688x^2}{3125}+\frac{1728x^3}{15625}+\cdots$$

b)

$$\left|\frac{16}{25}x\right| < 1$$

$$|x| < \frac{25}{16}$$

$$\frac{-25}{16} < x < \frac{25}{16}$$

9) a)

$$px - 4 = 0$$

$$x = 4/p$$

b) $y = -5px + 17$

$(0,17)$

c) (i) $3 - 4x > -3$

a) $x < 3/2$

$4/p < 3/2$

$p > 8/3$

b) $5p > 4$

$p > 4/5$

$p > 0.8$

$(p > 8/3)$

(ii)

$-3 = 3 - 4x$

$x = 3/2$

$3/2 = 4/p$

$p = 8/3$

(iii) $3 - 4x < -3$

a) $x > 3/2$

$4/p > 3/2$

$p < 8/3$

b) $4 > 5p$

$p < 4/5$

$p < 0.8$

$(p < 0.8)$

d) $\pm 5(px-4)-3<2x+5$

$(+)$

$5px-23<2x+5$

$(5p-2)x<28$

$(-)$

$-5px+17<2x+5$

$12<x(5p+2)$

$$x>\frac{12}{(5p+12)}$$

10)

$$\int x(x^2+1)\,e^{x^2+1}dx$$

$$\int (x^2+1)\left(xe^{x^2+1}\right)dx$$

$$u=x^2+1\ \rightarrow\ \frac{du}{dx}=2x$$

$$\frac{dv}{dx}=xe^{x^2+1}\rightarrow V=\frac{e^{x^2+1}}{2}$$

$$\Rightarrow \frac{(x^2+1)e^{x^2+1}}{2}-\int \frac{2xe^{x^2+1}}{2}dx$$

$$\Rightarrow \frac{(x^2+1)e^{x^2+1}}{2}-\frac{\left(e^{x^2+1}\right)}{2}+C$$

$$\Rightarrow \frac{x^2e^{x^2+1}}{2}+C$$

b)

$$\int (3\theta+4\cos 3\theta)^2\,d\theta$$

$$\Rightarrow \int (9\theta^2+16cos^2 3\theta+24\theta\cos 3\theta)\,d\theta$$

$$\Rightarrow \int 9\theta^2+8\cos 6\theta+8+24(\theta\cos 3\theta)\,d\theta$$

$$\Rightarrow \int \Big(9\theta^2+8\cos 6\theta+8)d\theta+24\int \theta\cos 3\theta\Big)\,d\theta$$

$$u=\theta\rightarrow \frac{du}{d\theta}=1$$

$$\Rightarrow \frac{9\theta^3}{3} + \frac{8\sin 6\theta}{6} + 8\theta + 24\left[\frac{\theta\sin 3\theta}{3} + \frac{\cos 3\theta}{9}\right] + C \quad \frac{dv}{d\theta} = \cos 3\theta \rightarrow V = \frac{\sin 3\theta}{3}$$

$$\frac{\theta\sin 3\theta}{3} - \int \frac{\sin 3\theta}{3}\, d\theta$$

$$\Rightarrow \frac{\theta\sin 3\theta}{3} + \frac{\cos 3\theta}{9} + C$$

11) **a)** $t = 0$

$R = 75 + 25(1) = 100$

100,000 rabbits

b) $75 + 25e^{0.07t} = 30 + 40e^{0.14t}$

$45 + 25e^{0.07t} = 40 \times (e^{0.07t})^2$

take $e^{0.07t} = u$

$45 + 25u = 40u^2$

$40u^2 - 25u - 45 = 0$

$8u^2 - 5u - 9 = 0$

$u = 1.41$ or $u = -0.79$

$e^{0.07t} = 1.41$

$t = \frac{ln 1.41}{0.07} = 49$ years

c) $R = 75 + 25e^{0.07t}$

$\frac{dR}{dt} = 25 \times 0.07e^{0.07t}$

$= 25 \times 0.07e^{0.07\times 15}$

$= 5.0008$

$=$ 5 thousand rabbits per year

12) **a)** $\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2 \sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2 \sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3 \sin A - 4\sin^3 A$

b) $1 - \sin 3x = \cos^2 x$

$1 - (3 \sin x - 4\sin^3 x) = 1 - \sin^2 A$

$4\sin^3 x + \sin^2 x - 3 \sin x = 0$

$\sin x\, [4\sin^2 x + \sin x - 3] = 0$

$\sin x\, (4 \sin x - 3)(\sin x + 1) = 0$

$\sin x = 0$ or $\sin x = {}^3/_4$ or $\sin(x) = (-1)$

$x = -180, 0, 180$ $\quad x = 48.6°,$ $\quad x = -90, 270$

360 $\quad 131.4°$

13) **a)**

$ln(2^k + 1) - ln(2) = ln(2^{k+1} + 2) - ln(2^k + 1)$

$$ln\left(\frac{2^k + 1}{2}\right) = ln\left(\frac{2^{k+1} + 2}{2^k + 1}\right)$$

$$\frac{2^k + 1}{2} = \frac{2^{k+1} + 2}{2^k + 1}$$

$(2^k + 1)^2 = 4 \times 2^k + 4$

$(2^k + 1)^2 - 4(2^k + 1) = 0$

$(2^k + 1)[2^k + 1 - 4] = 0$

$2^k + 1 = 0$ or $2^k = 3$

$$k = \frac{ln3}{ln2}$$

b)

$$\sum_{n=3}^{\alpha} \cos(180n)^\circ \left({}^3/_4\right)^n$$

$$\Rightarrow\ -\left({}^3/_4\right)^3 + \left({}^3/_4\right)^4 - \left({}^3/_4\right)^5 + \ldots..$$

$$a = -\left({}^3/_4\right)^3 \qquad S_n = \frac{a}{1-r} = \frac{\left({}^{-3}/_4\right)^5}{1+{}^3/_4} = \frac{-27}{112}$$

$$r = \left({}^{-3}/_4\right)$$

14) **a)**

$$\frac{dv}{dt} = -k$$

$$v = {}^4/_3\,\pi r^3$$

$$\frac{dv}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} \times (-k) = -\left(\frac{k}{4\pi}\right) \times \frac{1}{r^2}$$

take ${}^k/_{4\pi} = \mu \quad \frac{dr}{dt} = \frac{-\mu}{r^2}$

b)

$$\int r^2 dr = \int -\mu dt$$

$${}^{r^3}/_3 = -\mu t + c$$

$$t = 0, \Rightarrow \frac{(30)^3}{3} = 0 + c \qquad t = 10 \ \Rightarrow \frac{(15)^3}{3} = -\mu(10) + 9000$$

$$c = 9000 \qquad \mu = 787.5$$

$$\left({}^{r^3}/_3 = -787.5t + 9000\right)$$

c) $0 \leq t \leq 11.42$ seconds

15) $x = 6\sin 4y$

a) $\dfrac{dx}{dy} = 6\cos 4y \times 4 = 24\cos 4y$

$\dfrac{dy}{dx} = \dfrac{1}{24\cos 4y}$ (at orign) $\dfrac{dy}{dx} = \dfrac{1}{24}$

b) (i) $x = 6\sin(4y)$

$= 6(4y)$

$x = 24y$

$y = {}^{x}/_{24}$

(ii) the small angle approximation result is consistent with the exact derivative at the origin.

c) $\sin 4y = {}^{x}/_{6}$

$\sin^2 4y = {}^{x^2}/_{36}$, $\cos^2 4y = 1 - {}^{x^2}/_{36}$

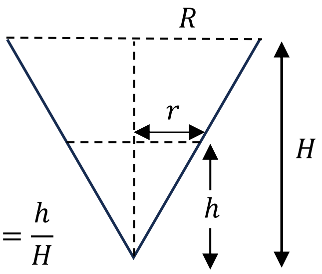
$= \dfrac{36 - x^2}{36}$

$$\frac{dy}{dx} = \frac{1}{24\left(\sqrt{\dfrac{36 - x^2}{36}}\right)} = \frac{1}{\left(\frac{24}{6}\sqrt{36 - x^2}\right)} = \frac{1}{4\sqrt{36 - x^2}}$$

16.

a)

$$\frac{dv}{dt} = -kv$$



$$v = {}^{1}/_{3}\,\pi r^2 h$$

$$v = {}^{1}/_{3}\,\pi \left(\frac{R}{H}\right)^2 h^3 \qquad \frac{r}{R} = \frac{h}{H}$$

$$\frac{dv}{dh} = \pi\left(\frac{R}{H}\right)^2 h^2 \qquad r = \left(\frac{R}{H}\right)^h$$

$$\frac{dh}{dt} = \frac{dh}{dv} \times \frac{dv}{dt} = \left(\frac{H^2}{R}\right)\frac{1}{\pi h^2} \times (-k) \times \frac{1}{3}(\pi)\left(\frac{R}{H}\right)^2 h^3$$

$$\frac{dh}{dt} = \left(-{}^{k}/_{3}\right)h \qquad \left(-{}^{k}/_{3}\right) = \mu$$

b)

$$\int \frac{1}{h}\,dh = \int -\mu\,dt \qquad \Rightarrow ln|h| - -\mu t + c$$

$$h = e^{-\mu t} \times e^{c}$$

$$h = Ae^{-\mu t}$$

$$\Rightarrow h = Ae^{kt}$$

(ii) $h = Ae^{kt}$

$$t = 0 \Rightarrow \quad 15 = A$$

$$t = 25 \;\Rightarrow 5 = 15e^{25k}$$

$$e^{25k} = {}^{1}/_{3}$$

$$k = {}^{1}/_{25} \;\; ln\left({}^{1}/_{3}\right)$$

c) $h = 15e^{-0.04\;ln(3)t}$

$$3 = 15e^{-0.04\;ln(3)t}$$

$$e^{0.04\;ln(3)t} = 5$$

$$0.04\;ln(3)\,t = ln5$$

$$t = \frac{1}{0.04} \times \frac{ln5}{ln3} = 36.6$$

A Level Maths Predicted Papers 2025 (June)

Paper 1: Pure Mathematics 1: Set 2

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This AL Maths paper 1 (Predicted Paper 2025: Set 2) has been created based on the most common topics from previous past papers. This paper should be excellent for helping students revise for exams; however, it should not be relied upon as the sole basis for revision.

Grade Boundary: A: 60% and A*: 80%

1.

Give that $(x - 2)$ is a factor of $g(x)$, find the values of the constant n

$$g(x) = 2x^3 + 5x^2 - n^2x + n$$

(3)

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2.

A cubic curve S is described by the equation:

$$f(x) = \ln(x - 2)$$

(a) Plot the graph of S. The sketch must clearly show all points where the curve intersects the coordinate axes.

(b) Hence draw separate graphs for the following equations, making sure to include the points where each graph meets the coordinate axes:

1. $y = |(f(x)|$

2. $y = 3f(2x) + 4$

3. $y = f(x + 1)$

(7)

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3.

The curve C has parametric equations

$$x = t^2 + 4t - 16 \qquad y = 10 \ln (t + 2) \qquad t > -2$$

(a) Show that a Cartesian equation for C is

$$y = P \ln (x + Q) \qquad x > -Q$$

where P and Q are integers to be found.

The curve C cuts the y-axis at the point X

(b) Show that the equation of the tangent to C at X can be written in the form

$$ax + by = c \ln \text{d}$$

where a, b, c, d are integers to be found.

(5)

4.

$$y = \sin 3x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\cos 3x$$

You may

- assume that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

5.

(i) Use proof by exhaustion to show that for $n \in \mathbb{N}$, $n < 5$

$$(n + 1)^3 > 3^n$$

(ii) Given that $p^3 + 7$ is odd, use proof by contradiction to show, using algebra, that p is even.

(5)

6. Given that

$g'(x) = 3x^3 + kx - 25$ where k is a constant

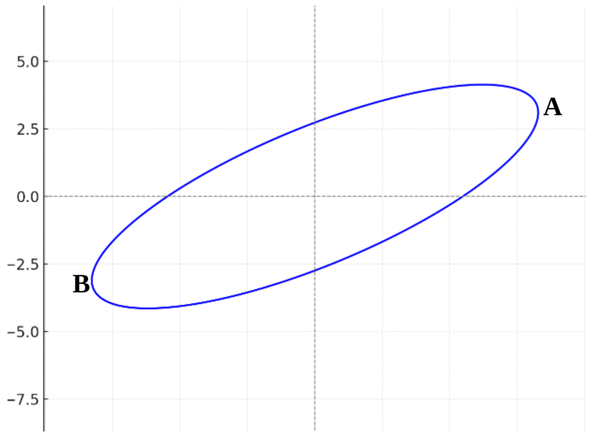
$(x - 3)$ is a factor of $g(x)$

the $g(x)$ intercept of C is 15

A curve R has equation $y = g(x)$, find, in simplest form, $g(x)$

(5)

7.



The diagram illustrates the curve given by the equation:

$$x^2 - 3xy + 4y^2 = 30$$

(a) Show that $\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$

The curve models the shape of a car racing track, with both x and y measured in km. Points A and B represent the locations on the track that are furthest west and east from the origin O, respectively.

(b) Using the result from part (a), find the exact coordinates of point A, the furthest East point and B the furthest west point.

(c) Briefly describe the method to find the coordinates of the point that is furthest north of the origin O. (You do not need to carry out the calculation)

(10)

8.

$$f(x) = 2\ln\left(\sqrt{2x-5}\right) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that f(x) = 0 has a root α in the interval [3.4, 4.3].

A student takes 4 as the first approximation to α.

Given f(4) = 3.099 and f$'$(4) = 16.67 to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(c) Show that α is the only two roots of f(x) = 0.

(6)

9.

(a) Determine the range of x values for which the curve $y = 12x^3 + 3x^2 + px + q$ is concave.

p and q are constant.

(b) Find the exact value of x for which

$$3(\ln 3x - 2)^3 - 12(\ln 3x - 2)^2 + 6\ln(3x - 2) = 24. \ x > 2/3$$

(6)

10.

show that

$$\lim_{\delta x \to 0} \sum_{x=2}^{5} \frac{7}{(5x+2)} = \ln m$$

where m is a constant to be found.

(4)

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11.

In the binomial expansion of

$(p + 3x)^7$ where p is a constant

the coefficient of x^4 is 1500

Find the value of p.

(3)

12.

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{44}P(22 - 4P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(a) determine the time taken, in years, for this population of meerkats to double,

(b) show that

$$P = \frac{N}{M + Re^{-At}}$$

where N, M and R are integers to be found.

(10)

13.

The circle C has equation

$$2x^2 + 2y^2 - 20x + 8y + 22 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

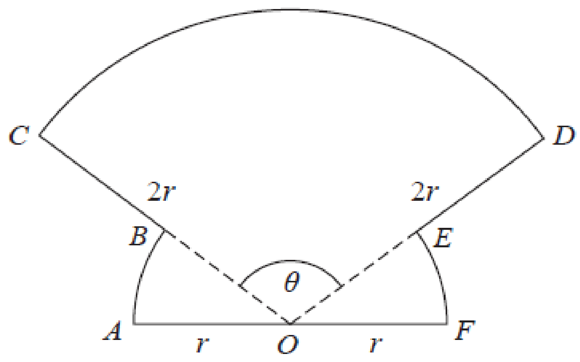
The line l has equation $y = 2x + p$ where p is a constant.

Given that l is a tangent to C,

(b) find the possible values of p, giving your answers as simplified surds.

(6)

14.



The shape *OABCDEFO* shown in Figure is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius 2*r*
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(*a*) write down, in terms of θ, the size of angle *AOB*

(*b*) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(*c*) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(7)

15.

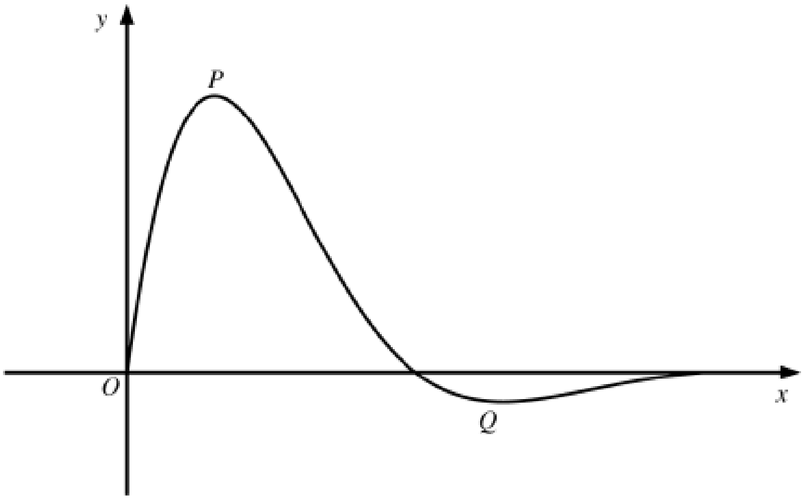


Figure shows a sketch of the curve with equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{8\sin x\cos x}{e^{(\sqrt{2})x-1}}, \quad 0 \leq x \leq \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure.

(*a*) Show that the x-coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(*b*) Using your answer to part (*a*), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{g}(3x)$

(ii) $y = 4 - 3\mathrm{g}(x)$

(5)

16.

A sequence $u_1, u_2, u_3 ...$ is defined by

$$u_1 = 34$$

$$u_{n+1} = u_n + 12\,sin\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(*a*) Find the value of u_2 u_3 and the value of u_4

Given that the sequence is periodic with order 4

(*b*) (i) write down the value of u_5

(ii) find the value of

$$\sum_{r=1}^{2025} u_r$$

(7)

17.

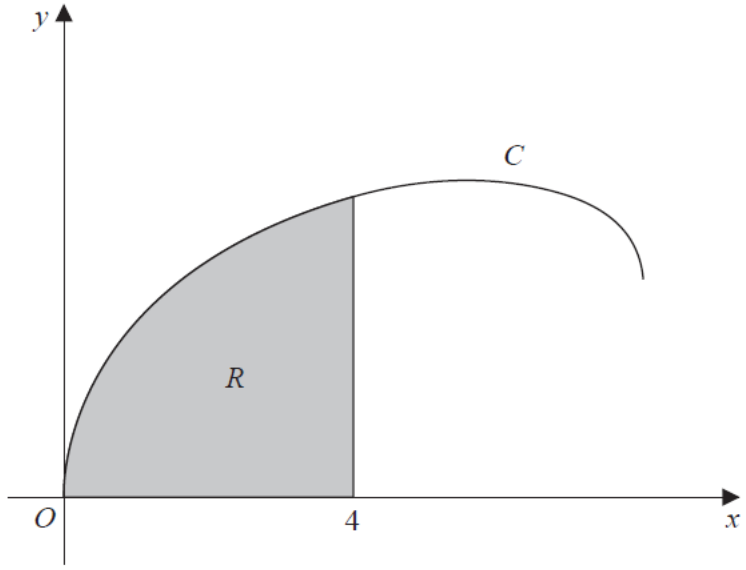


Figure shows a sketch of the curve C with parametric equations

$$x = 8\sin^2 t \qquad y = 4\sin t\cos t + 3\sin t \qquad 0 \leq t \leq \frac{\pi}{2}$$

The region R, shown shaded in Figure, is bounded by C, the x-axis and the line with equation $x = 4$

(*a*) Show that the area of R is given by

$$\int_0^a \left(8 - 8\cos 4t + 48\sin^2 t\cos t\right)\mathrm{d}t$$

where a is a constant to be found.

(*b*) Hence, using algebraic integration, find the exact area of R.

(10)

END

A Level Maths Predicted Papers 2025 (June)

Paper 1: Pure Mathematics 1: Set 2: Answers

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1.

$g(x) = 2x^3 + 5x^2 - n^2x + n$

$g(2) = 2(2)^3 + 5(2)^2 - n^2(2) + n = 0$

$16 + 20 - 2n^2 + n = 0$

$2n^2 - n - 36 = 0$

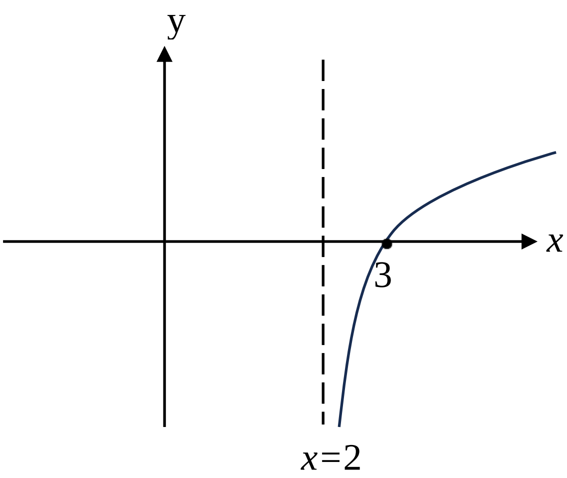
$(2n - 9)(n + 4) = 0$

$n = {}^{9}/_{2} \quad or \quad n = (-4)$

2.

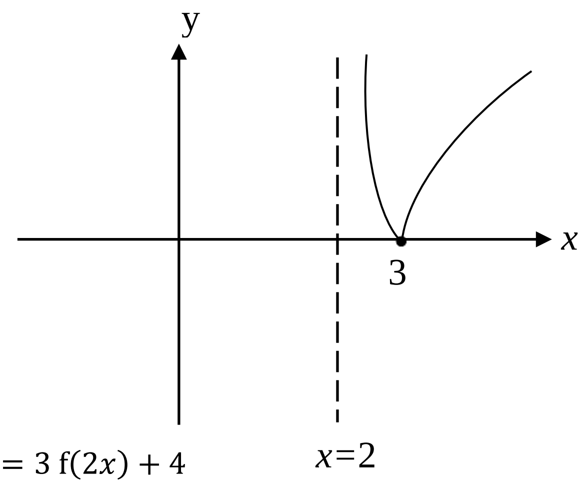
$\mathrm{f}(x) = ln\,(x - 2)$

a.

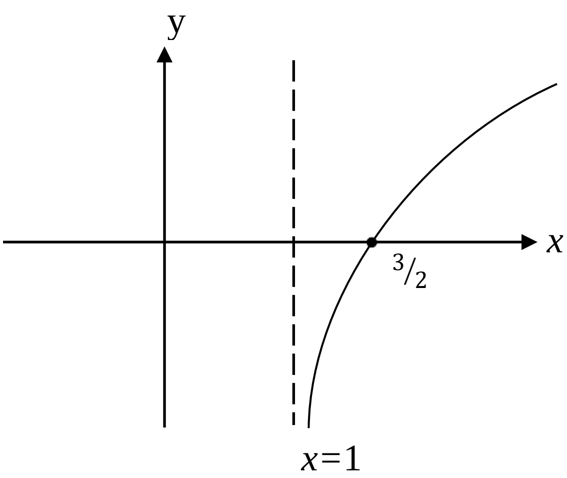


b.

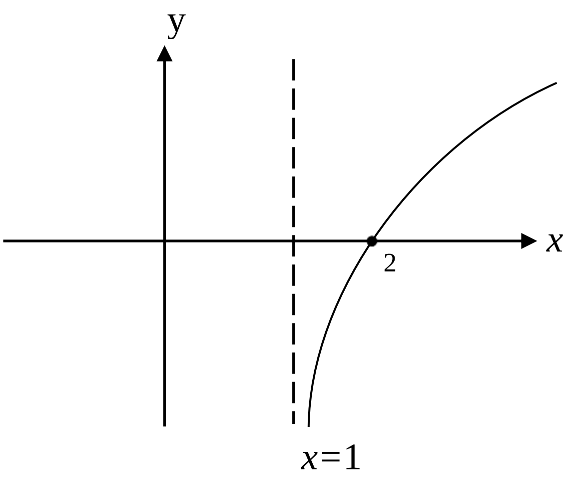
i. $y = |f(x)|$



ii. $y = 3\,f(2x) + 4$



iii. $y = f(x + 1)$



3. $x = t^2 + 4t - 16$ $\quad$ $y = 10ln\,(t + 2)$

a. $x = (t + 2)^2 - 2^2 - 16$ $\quad$ $ln(t + 2) = {}^{y}/_{10} \quad — ②$

$x = (t + 2)^2 - 20 \quad — ①$

$① \Rightarrow (t + 2)^2 = x + 20$

$② \Rightarrow {}^{1}/_{2}\, ln(x + 20) = {}^{y}/_{10}$

$5ln(t + 20) = y$

b.

$$\frac{dy}{dx} = 5\left(\frac{1}{x + 20}\right) \times 1 = \frac{5}{x + 20}$$

$$\frac{dy}{dx} = \frac{5}{x + 20}$$

At $x \rightarrow \quad x = 0$

$t^2 + 4t - 16 = 0$

$(t + 2)^2 - 20 = 0$

$(t + 2) = \pm\sqrt{20}$

$t = -2 \pm \sqrt{20}$

$t = -2 + \sqrt{20}$

$y = 10ln\,(-2 + \sqrt{20} + 2)$

$= 10ln(\sqrt{20})$

$y = 5ln20$

$$\frac{dy}{dx} = \frac{5}{0+20} = \frac{1}{4}$$

$y - 5ln20 = {}^{1}/_{4}\,(x - 0)$

$4y - 20ln20 - x = 0$

4. $y = \sin 3x$

$$f^1(x) \ = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(3x + 3h) - \sin 3x}{h}$$

$$= \lim_{h\to 0} \frac{\sin(3x)\cos(3h) + \cos(3x)\sin(3h) - \sin(3x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(3x)[\cos(3h) - 1] + \cos(3x)\sin(3h)}{h}$$

$$= \lim_{h\to 0} \sin(3x)\,(3)\left(\frac{\cos 3h - 1}{3h}\right) + \lim_{h\to 0} \cos 3x\,(3)\frac{\sin 3h}{3h}$$

$$\Rightarrow 0 + 3\cos 3x$$

$$= 3\cos 3x$$

5.

i. $n \in \mathbb{N} \ ; \ n < 5$

	L.H.S		R.H.S
$n = 4 \rightarrow$	$(4+1)^3$		3^4
	(125)	$>$	81
$n = 3 \rightarrow$	$(3+1)^3$		3^3
	64	$>$	27
$n = 2 \rightarrow$	$(2+1)^3$		3^2
	27	$>$	9
	$\vdots$		$\vdots$

There fore $n < 5$ for $n \in \mathbb{N}$

ii. $p^3 + 7 = 2k + 1(\text{odd})$

Assume $p = 2m + 1$ (odd)

$p^3 + 7 \Rightarrow ((2m) - 1)^3 + 7$

$= (2m)^3 + 3(2m)^2 + 3(2m) + 1 + 7$

$= 8m^3 + 12m^2 + 6m + 8$

$= 2[4m^3 + 6m^2 + 3m + 4]$

$=$ even

This contradicts the initial true therefore p is even

6.

$g^1(x) = 3x^3 + kx - 25$

$g(x) = \int 3x^3 + kx - 25$

$= \frac{3x^4}{4} + \frac{kn^2}{2} - 25 + c$

$g(3) = \frac{3}{4}(3)^4 + \frac{k}{2}(3)^2 - 25(2) + c = 0$

$\frac{3^5}{4} + \frac{9}{2}k - 50 + c = 0$ — 1

Intercept is 15 $\rightarrow c = 15$

1 $\Rightarrow \frac{243}{4} + \frac{9}{2}k - 50 + 15 = 0$

$k = {}^{-1}/_{6}$

$g(x) = 3x^3 {}^{-1}/_{6}\, x - 25$

7.

a. $x^2 - 3xy + 4y^2 = 30 \quad — ①$

$$2x - 3\left[x.\frac{dy}{dx} + y(1)\right] + 8y\frac{dy}{dx} = 0$$

$$2x - 3x\frac{dy}{dx} - 3y + 8y.\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(8y - 3x) = 3y - 2x$$

$$\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$$

b.

at A and B $\frac{dy}{dx} \to \alpha$

$$8y - 3x = 0$$

$$x = \frac{8}{3}y$$

$$① \Rightarrow \left(\frac{8}{3} - y\right)^2 - 3\left(\frac{8}{3}y\right)y + 4y^2 = 30$$

$$\frac{64}{9}y^2 - 8y^2 + 4y^2 = 30$$

$$\frac{28y^2}{9} = 30$$

$$y^2 = 9.64$$

$$y = \pm 3.1$$

at A

$y = 3.1$

$x = \frac{8}{3}(3.1)$

$x = 8.26$

at B

$y = -3.1$

$y = {}^{-8}/_{3}\,(3.1)$

$y = -8.26$

c. At further north $\frac{dy}{dx} = 0$, there fore $3y - 2x = 0$ we can find the answer.

8.

a. $f(x) = 2ln\left(\sqrt{2x-5}\right) + 2x^2 - 30$

$f(3.4) = -13.172$

$f(4.3) = 15.24$

Change of sign there fore solution is between 3.4 and 4.3

b.

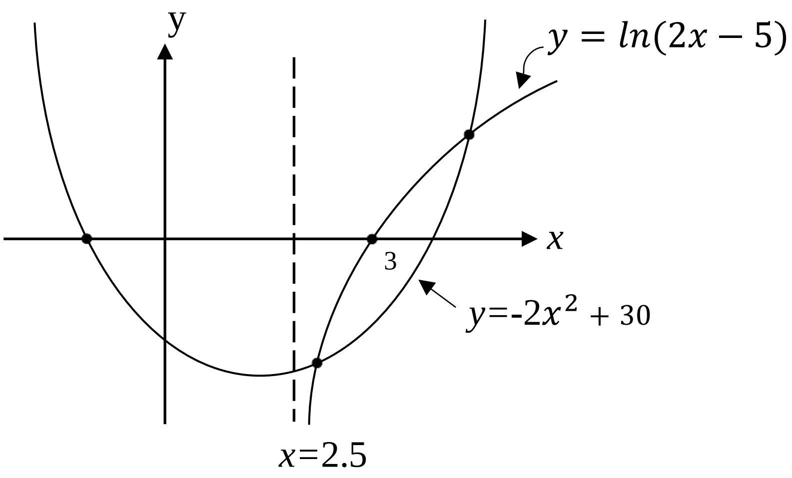
$$x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$$

$$x_1 = 4 - \frac{f(4)}{f^1(4)} = 4 - \frac{(3.099)}{(16.67)} = 3.814 = 3.81$$

c. $2ln\left(\sqrt{2x-5}\right) + 2x^2 - 30 = 0$

$ln\left(\sqrt{2x-5}\right) = -2x^2 + 30$

$ln\left(\sqrt{2x-5}\right) = -2(x^2 - 15)$



(2 intersection points)

9.

$y = 12x^3 + 3x^2 + px + q$

a. for concave

$$\frac{d^2y}{dx^2} < 0$$

$$\frac{dy}{dx} = 36x^2 + 6x + p$$

$$\frac{d^2y}{dx^2} = 72x + 6x < 0$$

$$x < \frac{-6}{72}$$

$$x < \frac{-1}{12}$$

b. $3(ln3x - 2)^3 - 12(ln(3n) - 2)^2 + 6(ln(3x - 2) = 24$

take $ln3x - 2 = u$

$3u^3 - 12u^2 + 6u - 24 = 0$

Take $(u) = 4$

$f(u) = 3(4)^3 - 12(3)^2 + 6(3) - 24 = 0$

$u - 4$ is a factor

$$
\begin{array}{r|l}
 & 3u^2 + 0 + 6 \\
\hline
u - 4 & 3u^3 - 12u^2 + 6u - 24 \\
 & \underline{3u^3 - 12u^2} \\
 & \qquad 0 + 6u \\
 & \qquad \underline{0 + 0} \\
 & \qquad\qquad 6u - 24 \\
 & \qquad\qquad \underline{6u - 24} \\
 & \qquad\qquad\qquad \underline{0}
\end{array}
$$

$f(x) = (u - 4)(3u^2 + 6)$

$ln(3x - 2) = 4$

$3x - 2 = e^4$ $\qquad$ $3u^2 + 6 \longrightarrow$ No solution

$x = \dfrac{e^4 + 2}{3}$

10.

$$\lim_{8x \to 0} \sum_{x=2}^{5} \frac{7}{(5x + 2)} = lnm$$

$$\Rightarrow \int_2^5 \frac{7}{(5x + 2)} dx \Rightarrow \left[\frac{7}{5} ln|5x + 2|\right]_2^5$$

$$= \tfrac{7}{5}[ln(27) - ln\,(12)]$$

$$= \tfrac{7}{5} ln\left(\tfrac{9}{4}\right)$$

11. $(p+3x)^7$

r^{th} term $\Rightarrow 7_{c_r}(p)^{7-r}(3x)^r$

x^4 term

$r = 4 \qquad 7_{c_4}(p)^3(3)^4 = 1500$

$$p^3 = \frac{1500}{7_{c_4} \times 3^4} = \frac{100}{189}$$

$$p = \sqrt[3]{\frac{100}{189}}$$

12.

a.

$$\frac{dp}{dt} = \frac{1}{44}\,p(22-4p)$$

$$\int \frac{dp}{p(22-4p)} = \int \frac{1}{44}dt$$

$$\frac{1}{P(11-2P)} = \frac{A}{p} + \frac{B}{11-2p}$$

$$1 = A(11-2p) + B(p)$$

$p = 0 \quad A = {}^1/_{11}$

$p = {}^{11}/_2 \qquad B = {}^2/_{11}$

$$\Rightarrow \int \left(\frac{{}^1/_{11}}{p} + \frac{\frac{2}{11}}{11-2p} \right) dp = \frac{1}{22}\int dt$$

$$\frac{1}{11}ln(p) + \frac{2}{11}\frac{ln(11-2p)}{(-2p)} = \frac{t}{22} + c$$

$$ln\left({}^p/_{11-2p}\right) = {}^t/_2 + k$$

$$\Rightarrow -ln\left(\frac{11-2p}{p}\right) = \frac{t}{2} + k$$

$$-ln({}^{11}/_p - 2) = {}^t/_2 + k$$

$$\frac{11}{p} - 2 = e^{-({}^{t}/_{2}+k)}$$

$$p = \frac{11}{2 + e^{-{}^{t}/_{2}} \times e^{-k}} = \frac{11}{2 + Ae^{-{}^{t}/_{2}}}$$

at $t = 0,$ $p = 1$

$$1 = \frac{11}{2 + A(e^{\circ})}$$

$$2 + A = 11$$

$$A = 9$$

$$P = \frac{11}{2 + 9e^{(-{}^{1}/_{2})t})}$$

b. Initially $p = 1$

now $p = 2$

$$2 = \frac{11}{2 + 9e^{-{}^{1}/_{2}t}}$$

$$4 + 18e^{-{}^{t}/_{2}} = 11 \qquad -{}^{t}/_{2} = ln({}^{7}/_{18})$$

$$e^{-{}^{t}/_{2}} = {}^{7}/_{18} \longrightarrow t = (-2)ln({}^{7}/_{18})$$

$$= 1.89 \text{ years}$$

13.

a. $x^2 + y^2 - 10x + 4y + 11 = 0$

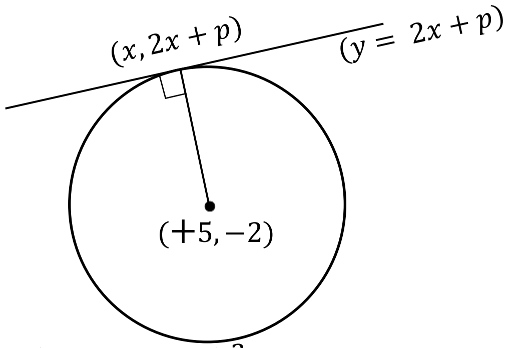
$(x-5)^2 - 5^2 + (y+2)^2 - 2^2 + 11 = 0$

$(x-5)^2 + (y+2)^2 = 18$

i. $(5, -2)$

ii. $\sqrt{18} \Rightarrow 3\sqrt{2}$

b.



$(x - 5)^2 + (2x + p + 2)^2 = 18$

$x^2 - 10x + 25 + 4x^2 + (p + 2)^2 + 4x(p + 2) = 18$

$5x^2 + x(4p - 2) + (7 + (p + 2)^2) = 0$

$b^2 - 4ac = 0$

$4(2p - 1)^2 - 4(5)\,(7 + (p + 2)^2) = 0$

$4p^2 - 4p + 1 - 35 - 5(p + 2)^2 = 0$

$4p^2 - 4p - 39 - 5p^2 - 20p - 20 = 0$

$-p^2 - 24p - 59 = 0$

$p^2 + 24p + 59 = 0$

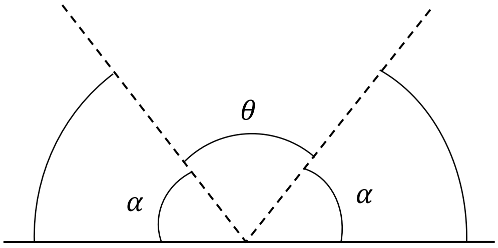
$p = -21.22 \quad or \quad p = -2.78$

$(-12 - \sqrt{85}) \quad (-12 + \sqrt{85})$

14.

a. $2\alpha + \theta = \pi$

$$\alpha = \frac{\pi - \theta}{2}$$



b.

$$\text{Area } \Rightarrow {}^{1}/_{2}\,(2r)^2(\theta) + 2\left[{}^{1}/_{2} \times r^2 \times \left(\frac{\pi - \theta}{2}\right)\right]$$

$$= {}^{1}/_{2}\,(4r^2\theta) + {}^{1}/_{2}\,\left(r^2(\pi - \theta)\right)$$

$$= {}^{1}/_{2}\,r^2[4\theta + \pi - \theta]$$

$$= {}^{1}/_{2}\,r^2(3\theta + \pi)$$

c.

Perimeter. Consider symmetric on one side.

$$p = r + r\left(\tfrac{\pi-\theta}{2}\right) + r + 2r\left({}^{\theta}/_{2}\right)$$

$$= r\left(1 + {}^{\pi}/_{2} - {}^{\theta}/_{2} + 1 + \theta\right)$$

$$= r\left(2 + {}^{\pi}/_{2} + {}^{\theta}/_{2}\right)$$

Total perimeter

$$\Rightarrow 2p$$

$$\Rightarrow 2p\left(2 + {}^{\pi}/_{2} + {}^{\theta}/_{2}\right) \Rightarrow r(4 + \pi + \theta)$$

15.

a. $g(x) = \frac{8\sin x - \cos x}{e^{(\sqrt{2})x-1}} = \frac{4\sin 2x}{e^{\sqrt{2x}-1}}$

$$g^1(x) = 4\left[\frac{e^{\sqrt{2x}-1} \times 2\cos(2x) - \sin(2x) \times e^{\sqrt{2x}-1} \times \sqrt{2}}{e^{2\sqrt{2x}-2}}\right]$$

$$= \frac{4e^{\sqrt{2x}-1}}{e^{2\sqrt{2x}-2}}\left[2\cos 2x - \sin 2x \times \sqrt{2}\right]$$

at turning points $g^1(x) = 0$

$$2\cos 2x - \sqrt{2}\sin 2x = 0$$

$$2 - \sqrt{2}\tan 2x = 0$$

$$\tan 2x = {}^{2}/_{\sqrt{2}} = \sqrt{2}$$

$$\tan 2x = \sqrt{2}$$

$$\tan 2x = \sqrt{2}$$

$$2x = \tan^{-1}(\sqrt{2}) = 0.9553, 4.0969$$

$$x = 0.47765, \ \ 2.04845$$

(max) (min)

b. i. $\left({}^{x}/_{3}\right)$ $\frac{2.04845}{3} \Rightarrow 0.6828 \leftarrow x$ coordinate

ii. $\left(4 - 3y\right)$ $0.47765 \leftarrow x$ coordinate

16.

a. $u_2 = 34 + 12\sin(\pi/_2) - 5(-1)^1$

$= 52$

$u_3 = 52 + 12\sin(\pi) - 5(-1)^2$

$= 47$

$u_4 = 47 + 12\sin(3\pi/_2) - 5(-1)^3$

$= 40$

b. $(\overline{34, 52, 47, 40}), (\overline{\quad\quad}) \ldots\ldots$

i. $u_5 = 34$

ii. $\dfrac{2025}{4} = 506\left(\dfrac{1}{4}\right) \longrightarrow \displaystyle\sum_{r=1}^{2025} u_r = 506(34 + 52 + 47 + 40) + 34)$

$= 87572$

17.

a. $R = \displaystyle\int_0^4 y.dx$

$x = 8\sin^2 t$

$\dfrac{dx}{dt} = 8 \times 2\sin t\,.\cos t$

$dx = 16\sin t\,.\cos t\,.dt$

$= \displaystyle\int_0^{\pi/_4} (4\sin t\cos t + 3\sin t) \times (16\sin t\cos t)dt$

$= \displaystyle\int_0^{\pi/_4} (64\sin^2 t\cos^2 t + 48\sin^2 t\cos t)\,dt$

$= \displaystyle\int_0^{\pi/_4} (16\sin^2 2t + 48\sin^2 t\cos t)\,dt$

$= \displaystyle\int_0^{\pi/_4} \left(16\frac{(1 - \cos 4t)}{2} + 48\sin^2 t\cos t\right)dt$

$= \displaystyle\int_0^{\pi/_4} (8 - 8\cos 4t + 48\sin^2 t\cos t)\,dt$

$x = 0$

$8\sin^2 t = 0$

$t = 0$

$x = 4$

$8\sin^2 t = 4$

$\sin^2 t = 1/_2 \qquad (> 0)$

$\sin t = 1/_{\sqrt{2}}$

$t = \pi/_4$

b. $\int_0^{\pi/4} (8 - 8\cos 4t + 48\cos t \sin^2 t)\,dt$

$= \left[8t - 8\left(\frac{\sin 4t}{4}\right) + 48\left(\frac{\sin^3 t}{3}\right)\right]_0^{\pi/4}$

$\Rightarrow (2\pi - 2(0) + 16\left(\frac{1}{2\sqrt{2}}\right) - (0)$

Area $\Rightarrow 2\pi + 4\sqrt{2}$