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A-LEVEL

MATHEMATICS

A Level Maths Predicted Paper 1
2026 (June)
Pure Mathematics 1 (Set 1)

Master AL Maths with clarity and confidence—
turning knowledge into real success.
A/Prof (Dr) Nathan.

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A Level Maths Predicted Paper 2026 (June)
Paper 1: Pure Mathematics 1 (Set 1)
Duration: 2 hours

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This AL Maths paper 1 (Predicted Paper June 2026: Set 1) has been created based on the most common topics from previous past papers. This paper should be excellent for helping students revise for exams; however, it should not be relied upon as the sole basis for revision

Grade Boundary: A: 61% and A*: 77%

1.

The curve $f(x)$ passes through the point (2, 3).

$$f'(x) = 3x^2 + 2x + 5$$

Find $f(x)$ in its simplest form.

(3)

2.

(a) For a small angle θ(in radians), show that

$$4\cos\ \theta + (\sin\ \theta + 1)^2 \approx 2\theta + 5.$$

(b) Hence determine an approximate solution of

$$4\cos\ \theta + (\sin\ \theta + 1)^2 \approx 32\tan\ \theta.$$

(3)

3.

For $x > 1$,

$$\frac{20x + 6 - 8x^2}{(1 - x)(2x + 1)} = A + \frac{B}{1 - x} + \frac{C}{2x + 1}.$$

(a) Find the values of A, B,and C.

Let

$$f(x) = \frac{20x + 6 - 8x^2}{(1 - x)(2x + 1)}.$$

(b) Prove that $f(x)$is an increasing function.

(6)

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4.

(a) Express $14\cos x - 8\sin x$ in the form

$$R\cos(x+\alpha)$$

where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$.

(b) Give full details of a sequence of two transformations needed to transform the curve

$$y = \sec x$$

onto the curve

$$y = \frac{25}{14\cos x - 8\sin x}$$

(7)

5.

$$g(x) = (x-3)(x+7)^2$$

(a) Sketch the curve $y = g(x)$, showing clearly the points where the curve meets the coordinate axes.

(b) A new curve has equation

$$y = g(x+n)$$

where $n$is a constant.

Given that this curve passes through the origin, find the two possible values of n.

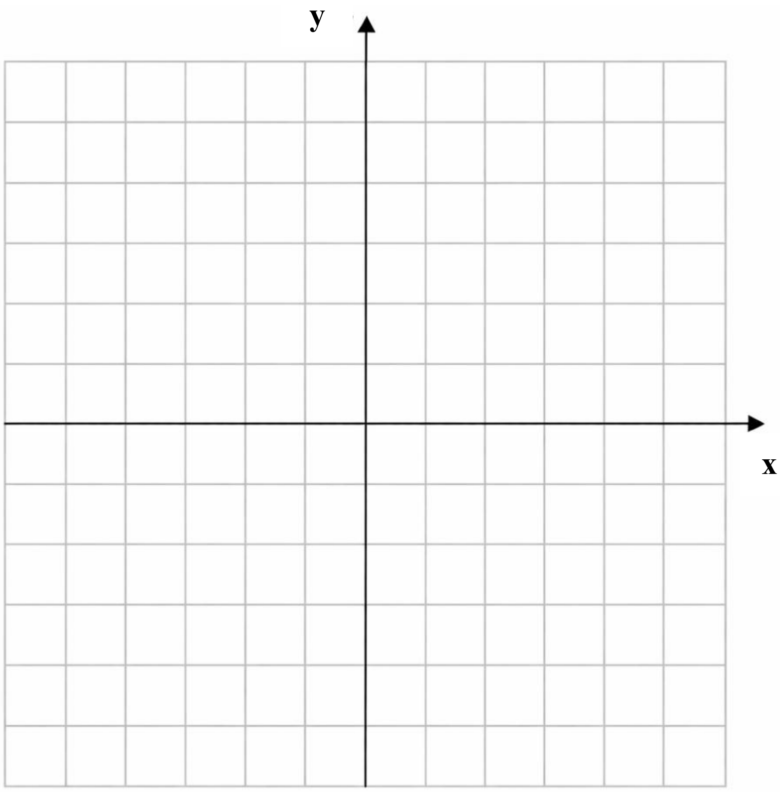
(c) Another curve has equation

$$y = g(x) + k$$

where $k$is a constant.

Given that this curve passes through the origin, find the possible value of k.

(6)



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6.

The function

$$g(x) = \frac{e^x}{3 - e^x}$$

is defined on the domain $x \in \mathbb{R}$, $x \neq \ln\ 3$.

(a) Find $g^{-1}(x)$

(b) Write down the range of $g^{-1}(x)$

(5)

7.

a) Use binomial expansions to show that

$$\sqrt{\frac{1+2x}{1-3x}} \approx 1+\frac{5}{2}x+\frac{35}{8}x^2$$

A student substitutes $x=\frac{1}{2}$ into both sides of the approximation in part (a) to try to obtain an approximation to $\sqrt{3}$.

(b) Give a reason why the student should not use $x=\frac{1}{2}$.

(c) Substitute $x=\frac{2}{11}$ into

$$\sqrt{\frac{1+2x}{1-3x}} \approx 1+\frac{5}{2}x+\frac{35}{8}x^2$$

to obtain an approximation to $\sqrt{3}$. Give your answer as a fraction in its simplest form.

(7)

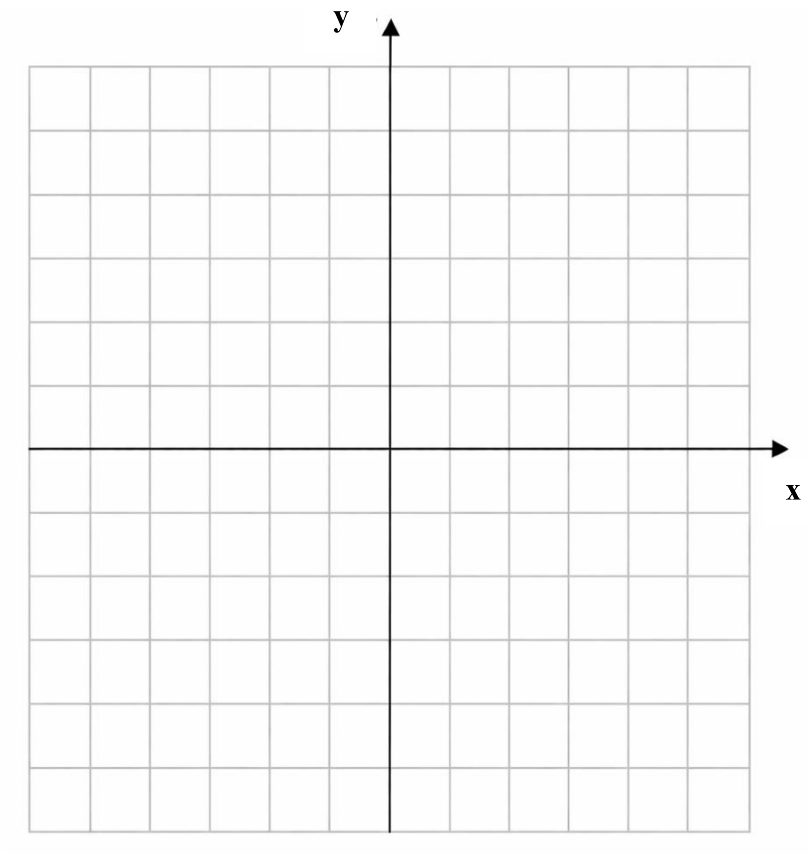
8. A curve C has parametric equations

$$x = 3 + 4\cos\ t, y = 2 + 5\cos\ 2t, 0 \leq t \leq 2\pi.$$

(a) Find an equation for C in the form $y = f(x)$ and state the domain for which the curve is defined.

(b) Sketch the curve.

(6)



9.

A circle C has centre $(10,6)$ and radius $2\sqrt{17}$.

A line L has equation $y = kx$.

(a) Show that the x-coordinate of any points of intersection of C and L satisfies the equation

$$(k^2 + 1)x^2 - (12k + 20)x + 68 = 0.$$

(b) Find the values of k for which the equation in part (a) has two distinct roots.

(c) Two lines are drawn from the origin which are tangents to C.

Find the coordinates of the points of contact between the tangents and C.

(7)

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10.

(a) show that

$$\lim_{\delta x \to 0} \sum_{x=0}^{2} \frac{x}{(5x^2+2)} = a \ln b$$

where a and b is a constant to be found.

(b) Evaluate:

$$\int (3\theta + 4\cos 3\,\theta)^2 \, d\theta$$

(7)

11.

(a) Sketch the graph of $y = g(x)$, where

$$g(x) = \begin{matrix} 8 - 2x, & x < 2, \\ (x-2)^2 + 4, & x \geq 2. \end{matrix}$$

(a) Find the value of $gg(1)$.

(b) Find all the values of x for which $g(x) > 12$.

The function h is defined as

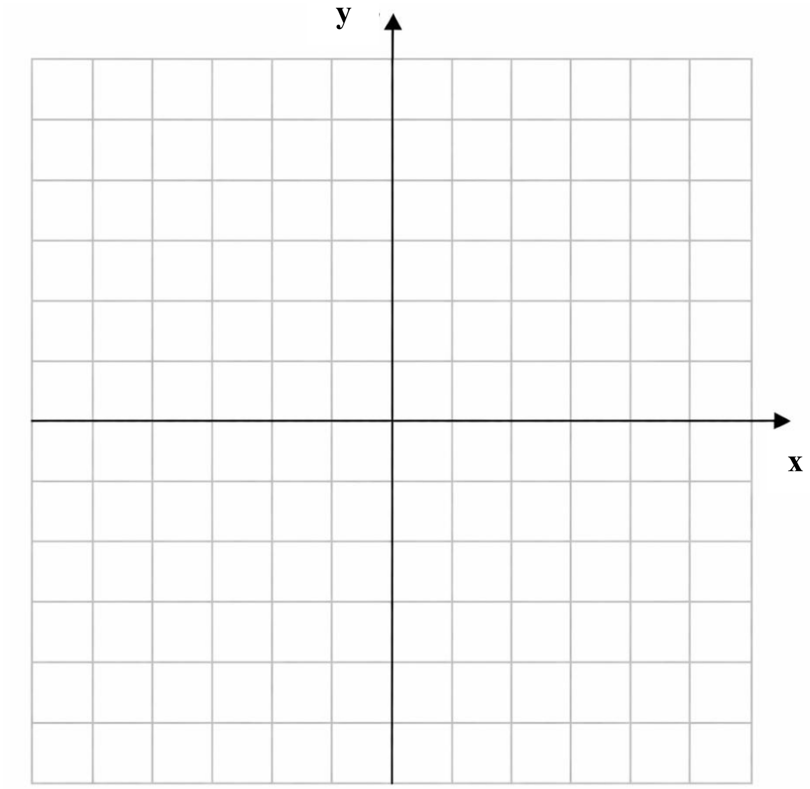
$$h(x) = (x-2)^2 + 4, x \geq 2.$$

(c) Explain why h has an inverse but g does not.

(d) Solve the equation

$$h^{-1}(x) = 2 + \sqrt{6}.$$

(8)



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12.

Find $\frac{dy}{dx}$

$$y = (\sin\ x)^x, 0 < x < \pi$$

(5)

13.

The value of a car is modelled using the formula

$$V = 25300e^{-0.2t} + 2500$$

where V is the value of the car and the car's age is t years.

(a) Find the initial value of the car.

Given the model predicts that the value of the car is decreasing at a rate of £1500 per year at the instant when $t = T$

(b) Show that

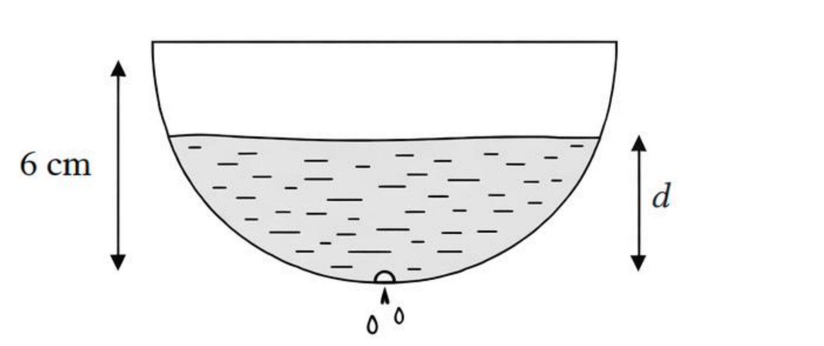
$$ke^{-0.2T} = 1500$$

k is constant

(c) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(5)

14.



A hemispherical bowl has radius 6 cm. Water leaks out through a small hole at the bottom. At time $t$minutes, the depth of water is $h$cm and the volume of water is V cm^3, where

$$V = \frac{\pi}{3}h^2(18 - h)(0 \leq h \leq 6).$$

In a model it is assumed that the rate at which the volume decreases is proportional to V, so

$$\frac{dV}{dt} = -kV,$$

where $k$is a positive constant.

(a) Show that

$$\frac{dh}{dt} = -\frac{k\,h(18 - h)}{3(12 - h)}.$$

(b) Express

$$\frac{3(12 - h)}{h(18 - h)}$$

in partial fractions.

(c) Given that when $t = 0$, $h = 6$, show that

$$h^2(18 - h) = 432e^{-kt}.$$

(d) Given also that when $t = 2$, $h = 5$, find k correct to 3 significant figures.

(10)

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15.

A curve, C, has the equation $10\sin\,y + 2x\cos\,y = Ax$

where A is a constant.

Cpasses through the point $P(\sqrt{3}, \frac{\pi}{3})$.

(a) Show that $A = 6$.

(b) Show that

$$\frac{dy}{dx} = \frac{3 - \cos\,y}{5\cos\,y - x\sin\,y}.$$

(c) Hence find the equation of the tangent to Cat P.

(5)

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16.

A spherical scoop of ice has initial radius 10 cm and is placed in a bowl.

In a model:

- the ice stays spherical as it melts
- t minutes after it is placed in the bowl, its radius is r cm
- the rate of decrease of the radius is inversely proportional to the square of the radius
- after 12 minutes, the radius is 7 cm

Using this model:

(a) Find an equation linking r and t.

(b) Find the time taken for the ice to melt completely.

(7)

17.

A student is investigating the following statement about natural numbers.

"$3n^3 - 3n$ is a multiple of 12"

(a) Prove, using algebra, that the statement is true for all odd numbers.

(b) Use a counter example to show that the statement is not always true.

(3)

END

TOTAL FOR PAPER IS 100 MARKS

A Level Maths Predicted Papers 2026 (June) Pure: Paper 1: Set 1: Answers

1.

$$f(x) = \int (3x^2 + 2x + 5)\,dx$$
$$f(x) = x^3 + x^2 + 5x + C$$

Since the curve passes through (2, 3):

$$3 = 2^3 + 2^2 + 5(2) + C$$
$$C = -19$$
$$\boxed{f(x) = x^3 + x^2 + 5x - 19}$$

2.

(a)

For small θ in radians, use the standard small-angle approximations:

$$\sin\,\theta \approx \theta, \cos\,\theta \approx 1 - \frac{\theta^2}{2}.$$

Now approximate the left-hand side:

$$4\cos\,\theta + (\sin\,\theta + 1)^2 \quad \approx 4(1 - \frac{\theta^2}{2}) + (\theta + 1)^2$$
$$= 4 - 2\theta^2 + (\theta^2 + 2\theta + 1)$$
$$= 4 - 2\theta^2 + \theta^2 + 2\theta + 1$$
$$= 5 + 2\theta - \theta^2.$$

For a small angle, θ^2 is much smaller than θ, so we neglect the $-\theta^2$ term:

$$4\cos\,\theta + (\sin\,\theta + 1)^2 \approx 5 + 2\theta.$$

(b)

$$4\cos\,\theta + (\sin\,\theta + 1)^2 \approx 32\tan\,\theta$$
$$\tan\,\theta \approx \theta.$$
$$5 + 2\theta \approx 32\theta.$$
$$\theta \approx \frac{5}{30} = \frac{1}{6} \text{ radians}$$

3.

a) Find A, B, C

Multiply both sides by $(1 - x)(2x + 1)$:

$$20x + 6 - 8x^2 = A(1 - x)(2x + 1) + B(2x + 1) + C(1 - x).$$
$$x = 1$$

Left-hand side (LHS):

$$20(1) + 6 - 8(1)^2 = 20 + 6 - 8 = 18.$$

Right-hand side (RHS):

$$B(2 \cdot 1 + 1) = 3B.$$
$$3B = 18 \Rightarrow B = 6.$$

$$x = -\frac{1}{2}$$

LHS:

$$20(-\frac{1}{2}) + 6 - 8(\frac{1}{4}) = -10 + 6 - 2 = -6.$$

RHS:

$$C(1 - (-\frac{1}{2})) = C(1 + \frac{1}{2}) = C \cdot \frac{3}{2}.$$

$$\frac{3}{2}C = -6 \Rightarrow C = -4.$$

$x = 0$
LHS:

$$20(0) + 6 - 8(0) = 6.$$

RHS:

$$A(1)(1) + B(1) + C(1) = A + B + C = A + 6 - 4 = A + 2.$$
$$A + 2 = 6 \Rightarrow A = 4.$$
$$\frac{20x + 6 - 8x^2}{(1-x)(2x+1)} = 4 + \frac{6}{1-x} - \frac{4}{2x+1}.$$

(b) Prove that $f(x)$is an increasing function

$$f(x) = \frac{20x + 6 - 8x^2}{(1-x)(2x+1)}, x > 1,$$
$$f(x) = 4 + \frac{6}{1-x} - \frac{4}{2x+1}.$$

To show $f(x)$is increasing, we show that its derivative $f'(x)$is positive for $x > 1$.
Differentiate $f(x)$

1. $\frac{d}{dx}(4) = 0.$
2. For $\frac{6}{1-x} = 6(1-x)^{-1}$:
$$\frac{d}{dx}[6(1-x)^{-1}] = 6 \cdot (-1) \cdot (1-x)^{-2} \cdot (-1) = \frac{6}{(1-x)^2}.$$
3. For $-\frac{4}{2x+1} = -4(2x+1)^{-1}$:
$$\frac{d}{dx}[-4(2x+1)^{-1}] = -4 \cdot (-1) \cdot (2x+1)^{-2} \cdot 2 = \frac{8}{(2x+1)^2}.$$

$$f'(x) = \frac{6}{(1-x)^2} + \frac{8}{(2x+1)^2}.$$

- $(1-x)^2 > 0$
- $(2x+1)^2 > 0$

$$\frac{6}{(1-x)^2} > 0, \frac{8}{(2x+1)^2} > 0.$$
$$f'(x) = \frac{6}{(1-x)^2} + \frac{8}{(2x+1)^2} > 0$$
$$\boxed{f(x) \text{ is an increasing function}}$$

4.

(a) We want

$$14\cos x - 8\sin x = R\cos(x + a),$$

with $R > 0$and $0 \leq a \leq \frac{\pi}{2}$.
Use

$$R\cos(x + a) = R(\cos x\cos a - \sin x\sin a) = (R\cos a)\cos x - (R\sin a)\sin x.$$

Match coefficients with $14\cos x - 8\sin x$:

$$R\cos a = 14, R\sin a = 8.$$
$$R = \sqrt{14^2 + 8^2} = \sqrt{196 + 64} = \sqrt{260} = 2\sqrt{65}.$$
$$\tan a = \frac{\sin a}{\cos a} = \frac{4}{7}, a = \arctan\left(\frac{4}{7}\right),$$

which lies in $[0, \pi/2]$.

$$\boxed{14\cos\ x - 8\sin\ x = 2\sqrt{65}\cos\left(x + \arctan\frac{4}{7}\right)}.$$

(b)

$$y = \frac{25}{14\cos\ x - 8\sin\ x}.$$

Using part (a),

$$14\cos\ x - 8\sin\ x = 2\sqrt{65}\cos(x + a), a = \arctan\frac{4}{7},$$

$$y = \frac{25}{2\sqrt{65}\cos(x+a)} = \frac{25}{2\sqrt{65}}\sec(x+a).$$

We want to describe two transformations that map $y = \sec\ x$onto $y = \frac{25}{14\cos\ x - 8\sin\ x}$.

$$y = \sec(x+a) \longrightarrow y = \frac{25}{2\sqrt{65}}\sec(x+a) = \frac{25}{14\cos\ x - 8\sin\ x}.$$

- translate the graph of $y = \sec x$ left by $\arctan(4/7)$;
- then apply a vertical stretch by factor $\frac{25}{2\sqrt{65}}$.

5.

(a) Sketch $y = g(x)$

$$g(x) = (x-3)(x+7)^2$$

x-intercepts

$g(x) = 0$:

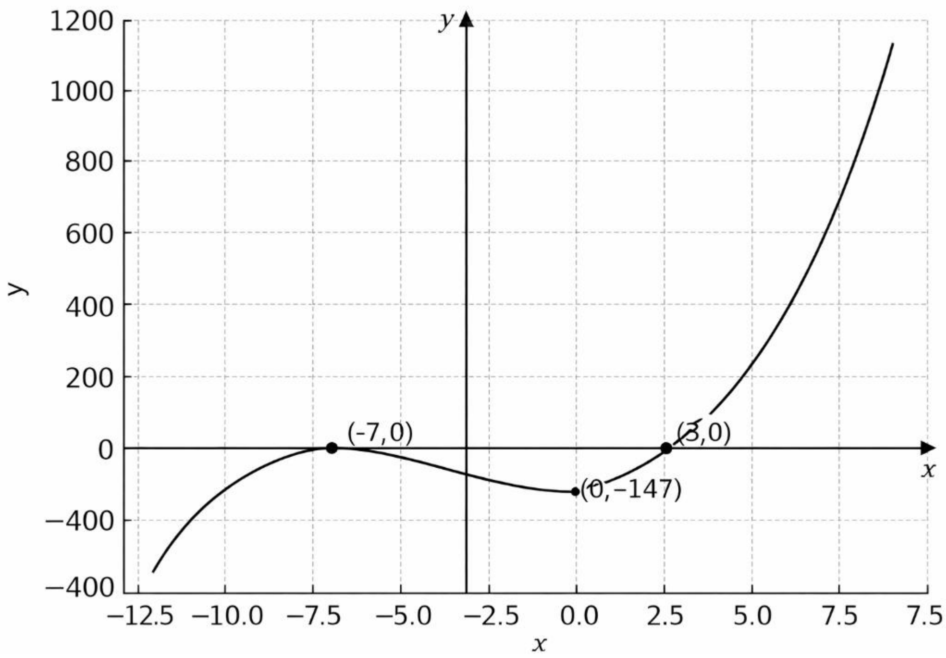
$$x = 3 \text{ or } x = -7$$

Since $(x+7)^2$is a repeated factor, the curve touches the x-axis at $x = -7$and turns there. At $x = 3$, the curve crosses the x-axis.

y-intercept

Set $x = 0$:

$$g(0) = (0-3)(0+7)^2$$
$$= -3 \times 49 = -147$$



(b) Find the two possible values of n

$$y = g(x+n)$$

passes through the origin, so when $x = 0$, $y = 0$.

$$g(n) = 0$$

$$g(n) = (n-3)(n+7)^2$$

$$\boxed{n = 3 \text{ or } n = -7}$$

(c) Find the possible value of k

The curve

$$y = g(x) + k$$

passes through the origin, so when $x = 0$, $y = 0$.

$$g(0) + k = 0$$

$$g(0) = -147$$

$$-147 + k = 0$$

$$\boxed{k = 147}$$

6.

(a) Find $g^{-1}(x)$

$$y = \frac{e^x}{3 - e^x}.$$

$$3y - ye^x = e^x.$$

$$e^x = \frac{3y}{1+y}.$$

$$x = \ln\left(\frac{3y}{1+y}\right).$$

$$\boxed{g^{-1}(x) = \ln\left(\frac{3x}{1+x}\right)}.$$

(Implicit condition: $\frac{3x}{1+x} > 0$and $x \neq -1$, so the logarithm is defined.)

(b) Range of $g^{-1}(x)$

The range of $g^{-1}$is the same as the domain of g.

The domain of $g$was given as

$$x \in \mathbb{R}, x \neq \ln 3,$$

$$(-\infty, \ln 3) \cup (\ln 3, \infty).$$

$$\boxed{\text{Range of } g^{-1}(x) = (-\infty, \ln 3) \cup (\ln 3, \infty).}$$

7. (a)

$$(1+u)^{\frac{1}{2}} \approx 1 + \frac{1}{2}u - \frac{1}{8}u^2, (1+u)^{-\frac{1}{2}} \approx 1 - \frac{1}{2}u + \frac{3}{8}u^2$$

(valid for $|u| < 1$).

$$(1+2x)^{1/2} \approx 1 + \frac{1}{2}(2x) - \frac{1}{8}(2x)^2 = 1 + x - \frac{1}{2}x^2$$

$$(1-3x)^{-1/2} = (1+(-3x))^{-1/2} \approx 1 - \frac{1}{2}(-3x) + \frac{3}{8}(-3x)^2 = 1 + \frac{3}{2}x + \frac{27}{8}x^2$$

(multiply, keep terms up to x^2):

$$\left(1 + x - \frac{1}{2}x^2\right)\left(1 + \frac{3}{2}x + \frac{27}{8}x^2\right)$$

Coefficient of x: $x + \frac{3}{2}x = \frac{5}{2}x$

Coefficient of x^2:

$$\frac{27}{8}+\frac{3}{2}-\frac{1}{2}=\frac{27}{8}+1=\frac{35}{8}$$

$$\boxed{\sqrt{\frac{1+2x}{1-3x}}\approx 1+\frac{5}{2}x+\frac{35}{8}x^2}$$

(b) Here we need both:

$$|\,2x\,|<1 \text{ and } |\,3x\,|<1$$
$$|\,x\,|<\frac{1}{3}$$

But $x=\frac{1}{2}$ does not satisfy this, so the expansion is not valid (and the approximation would be unreliable).

$$\boxed{\text{Because } |\,x\,|<\frac{1}{3} \text{ is required, but } \frac{1}{2}>\frac{1}{3}.}$$

(c)

First note that if $x=\frac{2}{11}$,

$$\frac{1+2x}{1-3x}=\frac{1+\frac{4}{11}}{1-\frac{6}{11}}=\frac{\frac{15}{11}}{\frac{5}{11}}=3$$

so the left-hand side is $\sqrt{3}$.

Now substitute $x=\frac{2}{11}$into the approximation:

$$1+\frac{5}{2}\cdot\frac{2}{11}+\frac{35}{8}\left(\frac{2}{11}\right)^2=1+\frac{5}{11}+\frac{35}{8}\cdot\frac{4}{121}$$

$$\boxed{\sqrt{3}\approx\frac{387}{242}}$$

8.

(a) Cartesian equation $y=f(x)$and domain

From $x=3+4\cos\ t$,

$$\cos\ t=\frac{x-3}{4}.$$

Use $\cos\ 2t=2\cos^2 t-1$.

$$y=2+5\cos\ 2t=2+5(2\cos^2 t-1)=2+10\cos^2 t-5=10\cos^2 t-3.$$

Substitute $\cos\ t=\frac{x-3}{4}$:

$$\cos^2 t=\left(\frac{x-3}{4}\right)^2=\frac{(x-3)^2}{16},$$
$$y=10\cdot\frac{(x-3)^2}{16}-3=\frac{10}{16}(x-3)^2-3=\frac{5}{8}(x-3)^2-3.$$
$$\boxed{y=\frac{5}{8}(x-3)^2-3.}$$

Since $\cos\ t\in[-1,1]$,

$$x=3+4\cos\ t\in[3-4,3+4]=[-1,7].$$

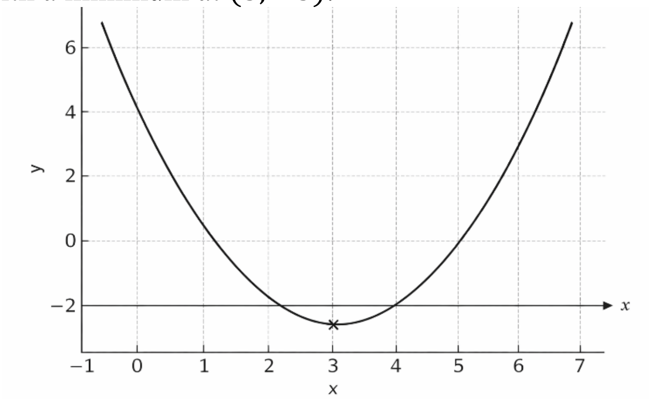
So the domain is

$$\boxed{-1 \le x \le 7.}$$

(b) Sketch of the curve

$$y = \frac{5}{8}(x-3)^2 - 3,$$

So you sketch the part of this parabola between $(-1,7)$and $(7,7)$, symmetric about the vertical line $x = 3$, with a minimum at $(3, -3)$.



9.

(a) Show that the x-coordinate satisfies $(k^2+1)x^2 - (12k+20)x + 68 = 0$

Substitute $y = kx$into the circle's equation:

$$(x-10)^2 + (kx-6)^2 = 68.$$

Expand:

$$x^2 - 20x + 100 + k^2x^2 - 12kx + 36 = 68.$$
$$(k^2+1)x^2 + (-20-12k)x + (100+36-68) = 0$$
$$\Rightarrow (k^2+1)x^2 - (12k+20)x + 68 = 0.$$

(b) Values of k for which the equation has two distinct roots

For the quadratic

$$(k^2+1)x^2 - (12k+20)x + 68 = 0,$$

we need the discriminant > 0.

$a = k^2 + 1$, $b = -(12k+20)$, $c = 68$.

$$\Delta = b^2 - 4ac = (12k+20)^2 - 4(k^2+1)\cdot 68.$$

Simplify:

$$\Delta = -128k^2 + 480k + 128 = -32(k-4)(4k+1).$$

For two distinct roots, $\Delta > 0$:

This is negative between the roots $k = -\frac{1}{4}$ and $k = 4$:

$$\boxed{-\frac{1}{4} < k < 4}.$$

(c) Points of contact of tangents from the origin

Tangents correspond to the limiting case where the quadratic has **equal** roots, i.e. $\Delta = 0$.

From part (b), this happens when

$$k = 4 \text{ or } k = -\frac{1}{4}.$$

So the two tangent lines from the origin are:

1. $y = 4x$
2. $y = -\frac{1}{4}x$

Now find their points of contact with the circle.

For $k = 4$:

Use the quadratic from (a) with $k = 4$:

$$(4^2+1)x^2 - (12\cdot 4 + 20)x + 68 = 0$$

$$17x^2 - 68x + 68 = 0 \Rightarrow x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0 \Rightarrow x = 2.$$

Then $y = 4x = 8$.

Point of contact: $\boxed{(2,8)}$.

For $k = -\frac{1}{4}$:

$$((-\frac{1}{4})^2 + 1)x^2 - (12 \cdot (-\frac{1}{4}) + 20)x + 68 = 0$$
$$(\frac{1}{16} + 1)x^2 - (-3 + 20)x + 68 = 0$$
$$17x^2 - 272x + 1088 = 0 \Rightarrow x^2 - 16x + 64 = 0$$
$$(x-8)^2 = 0 \Rightarrow x = 8.$$

Then $y = -\frac{1}{4}x = -2$.

Point of contact: $\boxed{(8,-2)}$.

So the coordinates of the points of contact of the tangents from the origin to the circle $C$are

$$\boxed{(2,8) \text{ and } (8,-2)}.$$

10.

$$\int_0^2 \frac{x}{5x^2+2}\,dx = [\frac{1}{10}\ln(5x^2+2)]_0^2 = \frac{1}{10}(\ln(5\times 2^2+2) - \ln(5\times 0^2+2))$$

$$\lim_{\Delta x \to 0} \sum \frac{x}{5x^2+2} = \frac{1}{10}\ln\ 11$$

$$\boxed{a = \frac{1}{10}, b = 11.}$$

b)

$$\int (3\theta + 4\cos 3\theta)^2\,d\theta$$
$$\Rightarrow \int (9\theta^2 + 16cos^2 3\theta + 24\theta\cos 3\theta)\,d\theta$$
$$\Rightarrow \int 9\theta^2 + 8\cos 6\theta + 8 + 24(\theta\cos 3\theta)\,d\theta$$
$$\Rightarrow \int \left(9\theta^2 + 8\cos 6\theta + 8)d\theta + 24\int \theta\cos 3\theta\right)d\theta$$

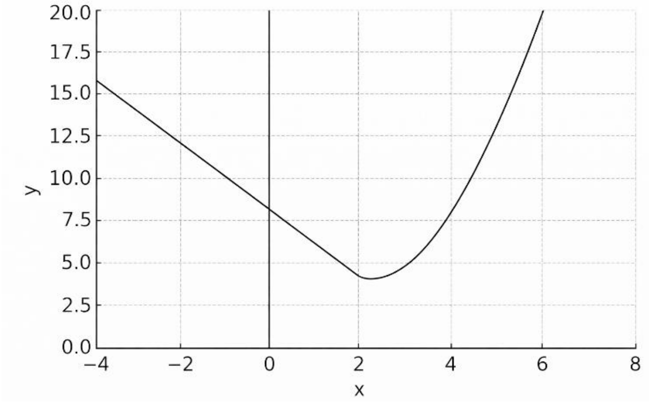
$$u = \theta \to \frac{du}{d\theta} = 1$$

$$\Rightarrow \frac{9\theta^3}{3} + \frac{8\sin 6\theta}{6} + 8\theta + 24\left[\frac{\theta\sin 3\theta}{3} + \frac{\cos 3\theta}{9}\right] + C$$

$$\frac{dv}{d\theta} = \cos 3\theta \to V = \frac{\sin 3\theta}{3}$$
$$\frac{\theta\sin 3\theta}{3} - \int \frac{\sin 3\theta}{3}d\theta$$
$$\Rightarrow \frac{\theta\sin 3\theta}{3} + \frac{\cos 3\theta}{9} + C$$

11.

(a)



(a) Find $gg(1)$

First find $g(1)$.

Since $1 < 2$, use the first rule:

$$g(1) = 8 - 2(1) = 8 - 2 = 6.$$

Now we need $g(g(1)) = g(6)$.

Since $6 \geq 2$, use the second rule:

$$g(6) = (6 - 2)^2 + 4 = 4^2 + 4 = 16 + 4 = 20.$$

$$\boxed{gg(1) = 20}$$

(b) Solve $g(x) > 12$

We consider each part of the piecewise function.

1. For $x < 2$: $g(x) = 8 - 2x$

Solve

$$8 - 2x > 12.$$
$$x < -2.$$

2. For $x \geq 2$: $g(x) = (x - 2)^2 + 4$

$$(x - 2)^2 + 4 > 12.$$
$$(x - 2)^2 > 8.$$
$$x > 2 + 2\sqrt{2} \text{ or } x < 2 - 2\sqrt{2}.$$

But for this branch we must have $x \geq 2$.

$$x > 2 + 2\sqrt{2}$$

Combine both parts

$$\boxed{x < -2 \text{ or } x > 2 + 2\sqrt{2}}.$$

c)

Therefore, $g$is not one-to-one and does not have an inverse on its whole domain.

(d) Solve $h^{-1}(x) = 2 + \sqrt{6}$

$$(x) = h(2 + \sqrt{6})$$
$$h(x) = (x - 2)^2 + 4, x \geq 2.$$
$$\boxed{x = 10}$$

12.

$$y = (\sin\ x)^x, 0 < x < \pi.$$

This requires logarithmic differentiation because both the base and exponent depend on x.

$$\ln\ y = x\ln\ (\sin\ x)$$
$$\frac{d}{dx}(\ln\ y) = \frac{1}{y}\frac{dy}{dx}$$

$$\frac{d}{dx}[x\ln(\sin x)] = 1 \cdot \ln(\sin x) + x \cdot \frac{1}{\sin x}\cos x$$
$$= \ln(\sin x) + x\cot x$$
$$\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + x\cot x$$
$$\boxed{\frac{dy}{dx} = (\sin x)^x(\ln(\sin x) + x\cot x)}$$

13.

(a) Initial value of the car

At $t = 0$,

$$V = 25300e^0 + 2500 = 25300(1) + 2500 = 27800.$$
$$\boxed{\text{Initial value } = £27\,800}$$

(b)(i) Show that $ke^{-0.2T} = 1500$

First find $\frac{dV}{dt}$:

$$V = 25300e^{-0.2t} + 2500$$

Differentiate:

$$\frac{dV}{dt} = 25300(-0.2)e^{-0.2t} + 0 = -5060e^{-0.2t}.$$

We are told that at $t = T$ the value is decreasing at £1500 per year, i.e.

$$\frac{dV}{dt} = -1500 \text{ when } t = T.$$
$$-5060e^{-0.2T} = -1500$$
$$5060e^{-0.2T} = 1500.$$

This has the required form $ke^{-0.2T} = 1500$with

$$\boxed{k = 5060}.$$

(b)(ii) Find the age of the car at this instant

From

$$5060e^{-0.2T} = 1500$$
$$e^{-0.2T} = \frac{1500}{5060}.$$
$$-0.2T = \ln\left(\frac{1500}{5060}\right)$$
$$T \approx 6.08 \text{ years.}$$

14.

(a) Differentiate $V = \frac{\pi}{3}(18h^2 - h^3)$:

$$\frac{dV}{dh} = \frac{\pi}{3}(36h - 3h^2) = \pi h(12 - h).$$

Chain rule:

$$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = \pi h(12 - h)\frac{dh}{dt}.$$

But $\frac{dV}{dt} = -kV = -k \cdot \frac{\pi}{3}h^2(18 - h)$.

$$\pi h(12 - h)\frac{dh}{dt} = -\frac{k\pi}{3}h^2(18 - h)$$
$$(12 - h)\frac{dh}{dt} = -\frac{k}{3}h(18 - h)$$

$$\boxed{\frac{dh}{dt}=-\frac{k\,h(18-h)}{3(12-h)}}.$$

(b)

$$\frac{3(12-h)}{h(18-h)}=\frac{A}{h}+\frac{B}{18-h}.$$

$$3(12-h)=A(18-h)+Bh.$$

Put $h=0$: $36=18A\Rightarrow A=2$.

Put $h=18$: $-18=18B\Rightarrow B=-1$.

$$\boxed{\frac{3(12-h)}{h(18-h)}=\frac{2}{h}-\frac{1}{18-h}}.$$

(c)

From (a):

$$\frac{dh}{dt}=-\frac{k\,h(18-h)}{3(12-h)}\Rightarrow\frac{3(12-h)}{h(18-h)}\,dh=-k\,dt.$$

Using (b):

$$(\frac{2}{h}-\frac{1}{18-h})dh=-k\,dt.$$

Integrate:

$$2\ln\ h+\ln\,(18-h)=-kt+C$$
$$\ln\ (h^2(18-h))=-kt+C$$
$$h^2(18-h)=Ce^{-kt}.$$

Use $t=0,h=6m$

$$C=6^2(18-6)=36\cdot 12=432.$$

$$\boxed{h^2(18-h)=432e^{-kt}}.$$

(d) When $t=2$, h = 5m

$$5^2(18-5)=432e^{-2k}\Rightarrow 25\cdot 13=432e^{-2k}\Rightarrow 325=432e^{-2k}.$$

$$e^{-2k}=\frac{325}{432}\Rightarrow -2k=\ln\,(\frac{325}{432})\Rightarrow k=\frac{1}{2}\ln\ (\frac{432}{325}).$$

$$k=0.142$$

15.

(a) Show that $A=6$

Substitute the coordinates of Pinto the equation.

At P: $x=\sqrt{3}$, $y=\frac{\pi}{3}$.

$$10\sin\,(\frac{\pi}{3})+2x\cos\,(\frac{\pi}{3})=Ax$$

Use $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3}=\frac{1}{2}$:

$$10\cdot\frac{\sqrt{3}}{2}+2\sqrt{3}\cdot\frac{1}{2}=A\sqrt{3}$$
$$5\sqrt{3}+\sqrt{3}=A\sqrt{3}$$
$$\boxed{A=6}$$

(b) Show that $\frac{dy}{dx}=\frac{3-\cos\,y}{5\cos\,y-x\sin\,y}$

$$10\sin\ y+2x\cos\ y=Ax.$$

Differentiate implicitly with respect to x.

$$10\cos\ y\ \frac{dy}{dx}+[2\cos\ y-2x\sin\ y\ \frac{dy}{dx}]=A.$$

Group the $\frac{dy}{dx}$ terms:

$$(10\cos\ y-2x\sin\ y)\frac{dy}{dx}+2\cos\ y=A.$$

$$\frac{dy}{dx}=\frac{6-2\cos\ y}{10\cos\ y-2x\sin\ y}$$

$$\frac{dy}{dx}=\frac{2(3-\cos\ y)}{2(5\cos\ y-x\sin\ y)}=\frac{3-\cos\ y}{5\cos\ y-x\sin\ y}.$$

(c) Equation of the tangent to Cat P

$$m=\frac{dy}{dx}|_P=\frac{3-\cos\ y}{5\cos\ y-x\sin\ y}=\frac{3-\frac{1}{2}}{5\cdot\frac{1}{2}-\sqrt{3}\cdot\frac{\sqrt{3}}{2}}.$$

$$m=\frac{5/2}{1}=\frac{5}{2}.$$

The tangent line at $P(\sqrt{3},\pi/3)$is

$$y-\frac{\pi}{3}=\frac{5}{2}(x-\sqrt{3})$$

$$y=\frac{5}{2}x+\frac{\pi}{3}-\sqrt{3}$$

16.

(a) Equation linking $r$and t

"Inversely proportional to r^2" means

$$\frac{dr}{dt}=-\frac{k}{r^2}(k>0)$$

$$r^2\ dr=-k\ dt$$

$$\int r^2\ dr=\int\ -k\ dt\Rightarrow\frac{r^3}{3}=-kt+C$$

$$r^3=-3kt+C'$$

Use $t=0, r=10$:

$$10^3=C'\Rightarrow C'=1000$$

$$r^3=1000-3kt$$

Use $t=12$, r=7

$$7^3=1000-36k\Rightarrow 343=1000-36k\Rightarrow 36k=657\Rightarrow k=\frac{657}{36}=\frac{73}{4}$$

$$r^3=1000-3(\frac{73}{4})t=1000-\frac{219}{4}t$$

$$\boxed{r^3=1000-\frac{219}{4}t\text{ or }r=(1000-\frac{219}{4}t)^{1/3}}$$

(b) Time to melt completely

Melts completely when $r=0$:

$$0=1000-\frac{219}{4}t\Rightarrow\frac{219}{4}t=1000\Rightarrow t=\frac{4000}{219}\approx 18.26$$

$$t = \frac{4000}{219} \text{ min} \approx 18.3 \text{ min}$$

17.

(a) Prove it is true for all odd numbers using $n = 2k + 1$

Let $n$be an odd natural number.

Then there exists an integer $k$such that

$$n = 2k + 1.$$

Now substitute this into $3n^3 - 3n$:

$$3n^3 - 3n = 3(2k + 1)^3 - 3(2k + 1).$$

First expand $(2k + 1)^3$:

$$(2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1.$$

$$3n^3 - 3n = 3(8k^3 + 12k^2 + 6k + 1) - 3(2k + 1).$$

$$= 24k^3 + 36k^2 + 18k + 3 - 6k - 3 = 24k^3 + 36k^2 + 12k.$$

$$24k^3 + 36k^2 + 12k = 12(2k^3 + 3k^2 + k).$$

it is a multiple of 12.

(b) Counterexample (show it is not always true)

Now choose a natural number that is not odd, for example $n = 2$.

Compute:

$$3n^3 - 3n = 3(2^3) - 3(2) = 3 \cdot 8 - 6 = 24 - 6 = 18.$$

But 18is not a multiple of 12 (since $18/12 = 1.5$).

is not always true.

END

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A-LEVEL

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Grade Boundary: A: 62% and A*: 79%

1.

Determine the range of x values for which the curve $y = 7x^3 + 3x^2 + x + 30$ is convex.

(3)

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2.

Solve

$$\frac{24}{x-1} \leq 6$$

$$x \neq 0$$

(3)

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3.

The curve has equation $y = 4 \cdot 2^{-x}$. The point $P(n, 20)$lies on the curve.

(a) Use logarithms to find n

(b) The table shows corresponding values of x and y for $y = 4 \cdot 2^{-x}$.

x	-0.5	1	2.5	4.0	5.5	7
y	5.657	2.000	0.707	0.250	0.088	0.031

(i) Use the trapezium rule (using all the y-values) to estimate

$$\int_{-0.5}^{7} 4 \cdot 2^{-x} \, dx$$

(ii) Use your answer to (b) (i) to estimate

$$\int_{-0.5}^{7} 2^{-x} \, dx + \int_{-7}^{0.5} 2^{x} \, dx$$

(8)

4.

(a) Sketch the graph of the equation $y = |4x - 3a|$, where $a > 0$. state the coordinates of the points where the graph intersects the coordinate axes.

(b) Solve, in terms of a, the inequality:

$$|4x - 3a| \leq 2x + 2a$$

(c) Find, in terms of a, the range of possible values of $g(x)$, where:

$$g(x) = 6a - \left|\frac{2a}{3} - x\right|$$

Given that

$$|4x - 3a| \leq 2x + 2a$$

(7)

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5.

The mass m of a bacterial culture is measured over time t hours.

The rate at which the mass increases is proportional to the mass present.

Write down a suitable equation relating m and t.

(2)

6.

$$f(x) = -3x^3 - 4x^2 - x - 10$$

(a) When $f(x)$ is divided by $(x-1)$ the remainder is R and the quotient is $Q(x)$.
Find $Q(x)$ and R.

(b) Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$. Hence prove, using algebra, that the equation $f(x) = 0$ has only one real solution.

(c) Find the range of values of x for which $f(x)$ is decreasing.

(7)

7.

$$\int_0^{\frac{\pi}{2}} \frac{2\sin\ 2\theta}{\sqrt{3+\cos\ \theta}}\, d\theta = a\sqrt{3} + b$$

where $a$and $b$are constants to be found.

(5)

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8.

In the binomial expansion of

$$(a + 3x)^6 \text{ where } a \text{ is a constant,}$$

the coefficient of x^2 is 1215.

Find the values of a.

(3)

9.

Using the first principle of differentiation, prove that if

$$y = \frac{2026}{x^3},$$

then

$$\frac{dy}{dx} = -\frac{6078}{x^4}.$$

(5)

10.

Use prof by contradiction to prove that for all x with $90° < x < 180°$,

$$\left|\frac{2\cos\ 2x}{\cos\ x - \sin\ x}\right| < 2.$$

(5)

11.

(i) Geometric series

A geometric series has first term $a$and common ratio $r (r \neq 1)$.

Prove that the sum of the first $n$terms, S_n, is

$$S_n = \frac{a(1 - r^n)}{1 - r}.$$

(ii) A liquid is stored in barrels.

Due to evaporation, the volume in any barrel at the end of each year is 12% less than the volume at the start of that year.

At the start of the first year, a barrel is filled with 180 litres.

(a) Show that the amount of liquid in the barrel at the end of 4 years is approximately 108 litres.

At the start of each year, a new barrel is filled with 180 litres, so that at the end of 30 years there are 30 barrels containing liquid.

(b) Calculate the total amount of liquid, to the nearest litre, in the 30 barrels at the end of 30 years.

(8)

12.

(i) Solve

$$2\log_3(4x+5) - \log_3(x+3) = 2$$

(ii) Given that $a, b > 0$

and

$$\log_{10} a + \log_{10} b = \log_{10}(a+b)$$

(a) Prove that

$$a = \frac{b}{b-1}$$

(b) Hence write down the full restriction on the value of b, giving a reason for your answer.

(6)

13.

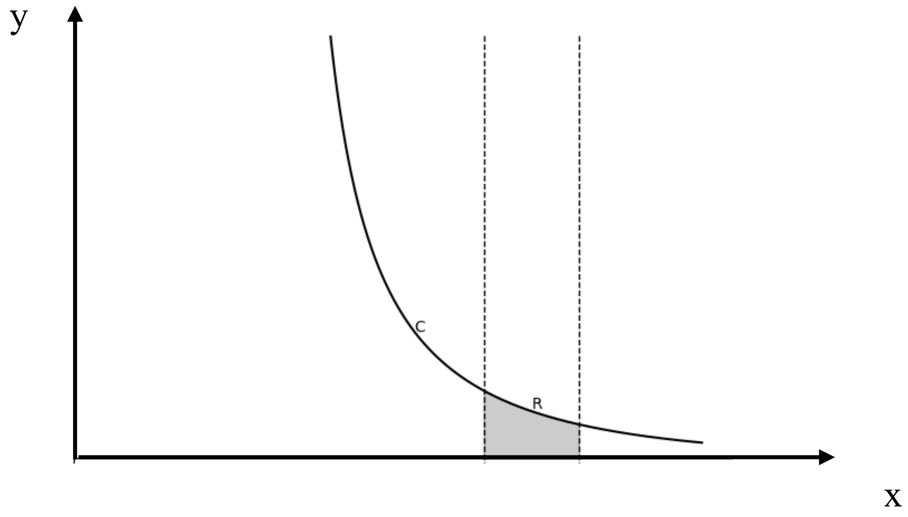

Figure shows a curve C with parametric equations

$$x = \ln(3t + 6), y = \frac{1}{t+1}, t > -1.$$

A point P lies on C.

Given that the gradient of C at P is -2,

(a) Use calculus to find the exact y-coordinate of P.

The region R is bounded by C, the x-axis, and the vertical lines

$$x = \ln 6 \text{ and } x = \ln\ 12$$

(b) (i) Show that the area of R can be written in the form

$$\int_a^b \frac{k}{(t+1)(t+2)}\, dt$$

where a, b, k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R

(11)

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14.

The curve C is given by $y = \ln(10 - x)$. The line is given by $y = x$ and They intersect at single point $x = a$

(a) Show that $2 < a < 3$

$$g(x) = xe^{-x^2} + 0.25$$

(b) Using $x_0 = -0.3$ as a first approximation, apply the Newton–Raphson method to $g(x)$ to obtain a third approximation. Give your answer to 3 decimal places.

(7)

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15.

(a) Prove the trigonometric identity:

$$sin3A = 3sinA - sin^3 A$$

(b) Using the result from part (a), solve the equation: $-180 \leq x \leq 360$

$$1 - sin3x = cos^2 x$$

(5)

16.

A tank initially contains 4 litres of water. Water starts to flow into the tank.

The rate at which the water flows into the tank is modelled by the differential equation

$$\frac{dV}{dt} = 10te^{-t}$$

where V litres is the volume of water in the tank, and $t$minutes is the time after water starts flowing in.

(a) Solve the differential equation and use the initial condition to find an equation for Vin terms of t.

The capacity of the tank is 15 litres.

(b) Determine, according to the model, whether or not the tank will ever become full. Give a reason.

(10)

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17.

(a) Find AB

Point A has position vector 3i+4j-5k

Point B has position vector 5i+7j-3k

Point C has position vector 8i+12j+8k

Wehere O is Origin

(b) Find the angle of ABC

(c) Show that quadrilateral OABC is trapezium

(5)

END

TOTAL FOR PAPER IS 100 MARKS

A Level Maths Predicted Paper 2: 2026 (June) Pure: Set 1: Answers

1.

$$y = 7x^3 + 3x^2 + x + 30$$
$$\frac{dy}{dx} = 21x^2 + 6x + 1$$
$$\frac{d^2y}{dx^2} = 42x + 6$$

The curve is convex when:

$$\frac{d^2y}{dx^2} > 0$$
$$42x + 6 > 0$$
$$42x > -6$$
$$x > -\frac{1}{7}$$

2.

Multiply both sides by $(x-1)^2$

Since $(x-1)^2 > 0$for all $x \neq 1$, the inequality sign does not change.

$$\frac{24}{x-1}(x-1)^2 \leq 6(x-1)^2$$
$$24(x-1) \leq 6(x-1)^2$$
$$0 \leq x^2 - 6x + 5$$
$$(x-1)(x-5) \geq 0$$
$$\boxed{x < 1 \text{ or } x \geq 5}$$

3.

Given $y = 4 \cdot 2^{-x}$and $P(n, 20)$lies on the curve:

$$20 = 4 \cdot 2^{-n}$$
$$5 = 2^{-n}$$
$$\ln 5 = \ln(2^{-n}) = -n\ln 2$$
$$n = -\frac{\ln 5}{\ln 2} \approx -\frac{1.6094}{0.6931} \approx -2.3219$$
$$\boxed{n \approx -2.32}$$

(b)

(i) Trapezium rule for $\int_{-0.5}^{7} 4 \cdot 2^{-x}\ dx$

The x-values are equally spaced:

$$h = 1.5$$

Trapeium rule:

$$\int_{-0.5}^{7} y\ dx \approx h(\frac{1}{2}y_0 + y_1 + y_2 + y_3 + y_4 + \frac{1}{2}y_5)$$

Using the table:

$$\approx 1.5(\frac{1}{2}(5.657) + 2.000 + 0.707 + 0.250 + 0.088 + \frac{1}{2}(0.031))$$
$$= 1.5(2.8285 + 2.000 + 0.707 + 0.250 + 0.088 + 0.0155)$$
$$= 1.5(5.889) = 8.8335$$
$$\boxed{\int_{-0.5}^{7} 4 \cdot 2^{-x}\, dx \approx 8.83}$$

(ii) Estimate $\int_{-0.5}^{7} 2^{-x}\, dx + \int_{-7}^{0.5} 2^{x}\, dx$

$$\int_{-0.5}^{7} 4 \cdot 2^{-x}\, dx \approx 8.83$$

$$\int_{-0.5}^{7} 2^{-x}\, dx \approx \frac{8.83}{4} = 2.2075$$

Let $x = -u$. Then when $x = -7$, $u = 7$; when $x = 0.5$, $u = -0.5$.

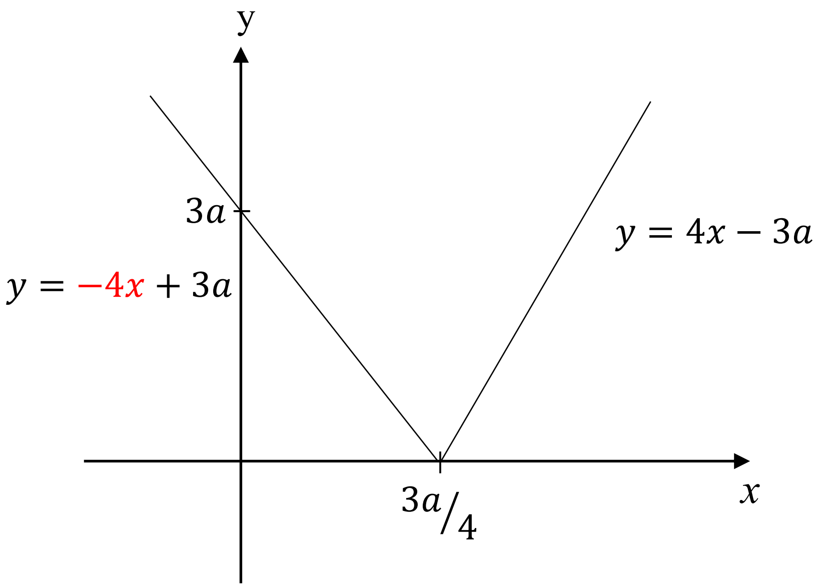
$$\int_{-7}^{0.5} 2^{x}\, dx = \int_{7}^{-0.5} 2^{-u}\,(-du) = \int_{-0.5}^{7} 2^{-u}\, du = \int_{-0.5}^{7} 2^{-x}\, dx$$

So the required sum is twice the first one:

$$\int_{-0.5}^{7} 2^{-x}\, dx + \int_{-7}^{0.5} 2^{x}\, dx \approx 2(2.2075) = 4.415$$
$$\boxed{\approx 4.42}$$

4.

a) $y = |4x - 3a|$



b) $-(4x - 3a) \leq 2x + 2a$ and $4x - 3a \leq 2x + 2a$

$6x \geq a$ $\qquad$ $2x \leq 5a$

$x \geq {}^{a}/_{6}$ $\qquad$ $x \leq {}^{5a}/_{2}$

c) $g(x) = 6a - \left|\frac{2a}{3} - x\right|$

$x \geq {}^{a}/_{6}$ $\qquad$ $x \leq {}^{5a}/_{2}$

$x = {}^{a}/_{6}$; $\quad 6a - |{}^{2a}/_{3} - {}^{a}/_{6}| = {}^{11a}/_{2}$

$x = {}^{5a}/_{2}$; $\quad 6a - |{}^{2a}/_{3} - {}^{5a}/_{2}| = {}^{25a}/_{6}$

$25a/6 \leq g(x) \leq 6a$

5.

Since the rate of increase of the mass is proportional to the mass present, the mass grows exponentially.

A suitable equation relating $m$and $t$is:

$$\boxed{m = Ae^{kt}}$$

6.

(a) Divide by $(x - 1)$

$$Q(x) = -3x^2 - 7x - 8, R = -18.$$

(b)(i) Factor theorem

Check $f(-2)$:

$$f(-2) = -3(-8) - 4(4) - (-2) - 10 = 24 - 16 + 2 - 10 = 0.$$

So $(x + 2)$is a factor of $f(x)$.

(b)(ii) Only one real solution

Divide $f(x)$by $(x + 2)$(or factor by inspection using the result above):

$$f(x) = (x + 2)(-3x^2 + 2x - 5).$$

$$-3x^2 + 2x - 5 = 0.$$

Its discriminant is

$$\Delta = 2^2 - 4(-3)(-5) = 4 - 60 = -56 < 0.$$

So $-3x^2 + 2x - 5$ has no real roots.

Therefore the only real root of $f(x) = 0$comes from $x + 2 = 0$, i.e.

$$\boxed{x = -2 \text{ is the only real solution.}}$$

(c)

$$f'(x) = -9x^2 - 8x - 1 = -(9x^2 + 8x + 1).$$

Solve $f'(x) = 0$:

$$9x^2 + 8x + 1 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 36}}{18} = \frac{-8 \pm \sqrt{28}}{18} = \frac{-4 \pm \sqrt{7}}{9}.$$

Since $f'(x) = -(9x^2 + 8x + 1)$, we have $f'(x) < 0$when $9x^2 + 8x + 1 > 0$, i.e. outside the roots.
So $f(x)$is decreasing for

$$\boxed{x < \frac{-4-\sqrt{7}}{9} \text{ or } x > \frac{-4+\sqrt{7}}{9}.}$$

7.

$$\int_0^{\pi/2} \frac{2\sin\ 2\theta}{\sqrt{3+\cos\ \theta}}\ d\theta = a\sqrt{3} + b$$

Use $\sin\ 2\theta = 2\sin\ \theta\cos\ \theta$

$$2\sin\ 2\theta = 4\sin\ \theta\cos\ \theta$$

So the integral becomes

$$\int_0^{\pi/2} \frac{4\sin\ \theta\cos\ \theta}{\sqrt{3+\cos\ \theta}}\ d\theta$$

Substitute $u = 3 + \cos\ \theta$

$$u = 3 + \cos\ \theta \Rightarrow du = -\sin\ \theta\, d\theta$$

Also $\cos\ \theta = u - 3$.
Then

$$\int \frac{4\sin\ \theta\cos\ \theta}{\sqrt{3+\cos\ \theta}}\ d\theta = -4\int \frac{u-3}{\sqrt{u}}\ du$$

$$= -4\int (u^{1/2} - 3u^{-1/2})\ du$$

$$-4(\frac{2}{3}u^{3/2} - 6u^{1/2}) = -\frac{8}{3}u^{3/2} + 24u^{1/2}$$

When $\theta = 0$, $\cos\ 0 = 1 \Rightarrow u = 4$
When $\theta = \pi/2$, $\cos\ (\pi/2) = 0 \Rightarrow u = 3$

$$[-\frac{8}{3}u^{3/2} + 24u^{1/2}]_4^3$$

$$\int_0^{\pi/2} \frac{2\sin\ 2\theta}{\sqrt{3+\cos\ \theta}}\ d\theta = 16\sqrt{3} - \frac{80}{3}$$

8.
For x^2, take $r = 2$:

$$\binom{6}{2}a^4(3^2)x^2$$

$$\binom{6}{2} \cdot a^4 \cdot 9 = 15 \cdot 9a^4 = 135a^4$$

$$135a^4 = 1215$$
$$a^4 = 9 \Rightarrow a = \sqrt[4]{9}$$
$$\boxed{a = \pm\sqrt{3}}$$

9.

Using first principles,

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$
$$f(x) = \frac{2026}{x^3},$$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\frac{2026}{(x+h)^3}-\frac{2026}{x^3}}{h}.$$
$$\frac{dy}{dx} = 2026\lim_{h\to 0}\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}.$$

$$\frac{1}{(x+h)^3}-\frac{1}{x^3} = \frac{x^3-(x+h)^3}{x^3(x+h)^3}.$$
$$\frac{dy}{dx} = 2026\lim_{h\to 0}\frac{x^3-(x+h)^3}{h\,x^3(x+h)^3}.$$
$$x^3-(x+h)^3 = x^3-(x^3+3x^2h+3xh^2+h^3)$$
$$= -3x^2h-3xh^2-h^3.$$
$$\frac{dy}{dx} = 2026\lim_{h\to 0}\frac{-3x^2h-3xh^2-h^3}{h\,x^3(x+h)^3}.$$
$$\frac{dy}{dx} = -2026\lim_{h\to 0}\frac{3x^2+3xh+h^2}{x^3(x+h)^3}.$$

Now let $h \to 0$:

$$\frac{dy}{dx} = -2026\cdot\frac{3x^2}{x^3\cdot x^3}.$$
$$= -2026\cdot\frac{3x^2}{x^6}.$$
$$= -\frac{6078}{x^4}.$$

10.

$$\left|\frac{2\cos\ 2x}{\cos\ x-\sin\ x}\right| < 2$$
$$\cos\ 2x = \cos^2 x-\sin^2 x = (\cos\ x-\sin\ x)(\cos\ x+\sin\ x).$$

So, for $90^\circ < x < 180^\circ$,

$$\frac{\cos\ 2x}{\cos\ x-\sin\ x} = \cos\ x+\sin\ x$$
$$|\cos\ x+\sin\ x| < 1.$$

Proof by contradiction

Assume the opposite, that for some $x$with $90^\circ < x < 180^\circ$,

$$|\cos x + \sin x| \geq 1.$$
$$(\cos x + \sin x)^2 \geq 1.$$
$$\cos^2 x + \sin^2 x + 2\sin x\cos x \geq 1.$$
$$1 + 2\sin x\cos x \geq 1$$
$$2\sin x\cos x \geq 0$$
$$\sin 2x \geq 0.$$

But if $90^\circ < x < 180^\circ$, then

$$180^\circ < 2x < 360^\circ,$$

and in this interval $\sin 2x < 0$(it is negative in quadrants III and IV, and only equals 0 at 180°or 360°, which are not included).

$$\sin 2x < 0.$$

we conclude:

$$\left|\frac{\cos 2x}{\cos x - \sin x}\right| < 1.$$

11.

(i) Proof of $S_n = \frac{a(1-r^n)}{1-r}$

The first $n$terms are

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1}.$$

Multiply by r:

$$rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n.$$

Subtract:

$$S_n - rS_n = a - ar^n$$
$$S_n(1-r) = a(1-r^n).$$

Since $r \neq 1$,

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}.$$

(ii)(a) Amount after 4 years

Each year the barrel keeps $100\% - 12\% = 88\% = 0.88$of its liquid.

So after 4 years:

$$V_4 = 180(0.88)^4.$$
$$V_4 = 180(0.59969536) = 107.9451648 \approx 108.$$
$$\boxed{V_4 \approx 108 \text{ litres.}}$$

(ii)(b) Total in 30 barrels after 30 years

At the end of 30 years:

- The newest barrel (filled at start of year 30) has had 1 year of evaporation: $180(0.88)^1$
- The next has had 2 years: $180(0.88)^2$
- …
- The oldest (filled at start of year 1) has had 30 years: $180(0.88)^{30}$

So total volume

$$T = 180(0.88 + 0.88^2 + \cdots + 0.88^{30}).$$

This is geometric with first term 180(0.88)and ratio 0.88, with 30terms:

$$T = 180(0.88) \cdot \frac{1 - (0.88)^{30}}{1 - 0.88}.$$

$$T = \frac{158.4(1 - 0.88^{30})}{0.12} = 1320(1 - 0.88^{30}).$$

$$\boxed{T \approx 1291 \text{ litres (nearest litre).}}$$

12.

(i) Solve $2\log_3(4x + 5) - \log_3(x + 3) = 2$

Domain:

$4x + 5 > 0 \Rightarrow x > -\frac{5}{4}$and $x + 3 > 0 \Rightarrow x > -3$.

So $x > -\frac{5}{4}$.

$$\log_3((4x + 5)^2) - \log_3(x + 3) = 2$$

$$\log_3(\frac{(4x + 5)^2}{x + 3}) = 2$$

$$\frac{(4x + 5)^2}{x + 3} = 3^2 = 9$$

$$(4x + 5)^2 = 9(x + 3)$$

$$16x^2 + 40x + 25 = 9x + 27$$

$$16x^2 + 31x - 2 = 0$$

$$\boxed{(16x - 1)(x + 2) = 0}$$

$$x = \frac{1}{16} \text{ or } x = -2$$

Check domain $x > -\frac{5}{4}$: $-2$is not allowed.

$$\boxed{x = \frac{1}{16}}$$

(ii) b)

$$\log_{10} a + \log_{10} b = \log_{10}(ab)$$

So the equation becomes:

$$\log_{10}(ab) = \log_{10}(a + b)$$

$$ab = a + b$$

$$ab - a = b$$

$$a(b - 1) = b$$

$$\boxed{a = \frac{b}{b - 1}}$$

(b) Restriction on b

We are told $b > 0$. Also $a > 0$and

$$a = \frac{b}{b-1}$$

For this to be defined:

$$b - 1 \neq 0 \Rightarrow b \neq 1$$

For $a > 0$:

$$\frac{b}{b-1} > 0$$

This is positive when numerator and denominator have the same sign.

Since $b > 0$, numerator is positive, so we need:

$$b - 1 > 0 \Rightarrow b > 1$$

$$\boxed{b > 1}$$

13.

(a) Find the exact y-coordinate of P

Differentiate with respect to t:

$$\frac{dx}{dt} = \frac{3}{3t+6} = \frac{1}{t+2}, \frac{dy}{dt} = -\frac{1}{(t+1)^2}.$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\frac{1}{(t+1)^2}}{\frac{1}{t+2}} = -\frac{t+2}{(t+1)^2}.$$

Given gradient is -2:

$$-\frac{t+2}{(t+1)^2} = -2 \Rightarrow \frac{t+2}{(t+1)^2} = 2.$$

$$t + 2 = 2(t+1)^2 = 2(t^2 + 2t + 1) = 2t^2 + 4t + 2$$

$$0 = 2t^2 + 3t$$

$$t(2t+3) = 0 \Rightarrow t = 0 \text{ or } t = -\frac{3}{2}.$$

But $t > -1$, so $t = -\frac{3}{2}$is invalid. Hence $t = 0$.

$$y = \frac{1}{t+1} = \frac{1}{1} = 1.$$

$$\boxed{\text{The exact } y\text{-coordinate of } P \text{ is } 1.}$$

(b)(i) Set up the area integral

Area under the curve between the two vertical lines is

$$A = \int y\, dx$$

Using the parameter t,

$$A = \int y \frac{dx}{dt}\, dt.$$

$$y = \frac{1}{t+1}, \frac{dx}{dt} = \frac{1}{t+2}.$$

$$y\frac{dx}{dt} = \frac{1}{(t+1)(t+2)}.$$

Now find t-limits from the given x-values:

- If $x = \ln\ 6$, then $\ln\,(3t+6) = \ln\ 6 \Rightarrow 3t + 6 = 6 \Rightarrow t = 0$.
- If $x = \ln\ 12$, then $\ln\,(3t+6) = \ln\ 12 \Rightarrow 3t + 6 = 12 \Rightarrow t = 2$.

Therefore

$$A = \int_0^2 \frac{1}{(t+1)(t+2)}\,dt.$$

So $a = 0,\ b = 2,\ k = 1$.

(b)(ii) Exact area

Use partial fractions:

$$\frac{1}{(t+1)(t+2)} = \frac{1}{t+1} - \frac{1}{t+2}.$$

Integrate:

$$A = \int_0^2 \left(\frac{1}{t+1} - \frac{1}{t+2}\right) dt = [\ln\,(t+1) - \ln\,(t+2)]_0^2.$$

$$A = (\ln\ 3 - \ln\ 4) - (\ln\ 1 - \ln\ 2) = \ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right) = \ln\left(\frac{3}{2}\right).$$

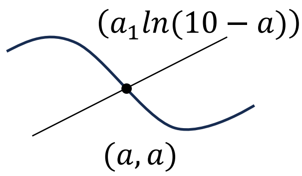
$$\boxed{\text{Area of } R = \ln\ \left(\frac{3}{2}\right)}$$

14.

(a)

$y = ln\,(10 - x)$

$a = \ln\,(10 - a)\ \ e^{a} = 10 - a$



$$ln(10 - x) = x$$
$$ln(10 - x) - x = 0$$

When $x = 2$

$$= ln(10 - 2) - 2$$
$$= ln(8) - 2$$
$$= 0.079$$

When $x = 3$

$$ln(10 - 3) - 3$$
$$\Rightarrow ln\,7 - 3$$
$$\Rightarrow -1.05$$

Since $f(2) > 0$and $f(3) < 0$, and $f$is continuous on $[2, 3]$, there is a root between 2 and 3. Therefore,

$$\boxed{2 < a < 3}$$

b) $g(x) = xe^{-x^2} + 0.25$

$$x_{n+1} = x_n - \frac{g_{(x)}}{g^1_{(x_n)}}$$

$$g_{(x)} = xe^{-x^2} + 0.25$$

$$g^1_{(x)} = 1.e^{-x^2} + x.e^{-x^2} \times (-2x) + 0$$

$$= e^{-x^2}(1 - 2x^2)$$

$$x_{n+1} = x_n - \frac{\left(x_n e^{-x_n^2} + 0.25\right)}{e^{-x_n^2}(1 - 2x_n^2)}$$

$x_0 = -0.3$

$$x_1 = x_0 - \frac{\left(x_0 e^{-x_n^2} + 0.25\right)}{e^{-x_n^2}\left(1 - 2x_n^2\right)}$$

$x_0 = -0.3$

$x_1 = -0.27$

$x_2 = -0.27$

$x_3 = -0.27$

Third approximation

$x = 0.269$

15.

a) $\sin(3A) = \sin(2A + A)$

$= \sin(2A).\cos A + \cos(2A).\sin A$

$= 2\sin A \cos A \cos A + (1 - 2\sin^2 A)\sin A$

$= 2\sin A(1 - \sin^2 A) + (1 - 2\sin^2 A)\sin A$

$= 3\sin A - 4\sin^3 A$

b) $1 - \sin 3x = \cos^2 x$

$1 - (3\sin x - 4\sin^3 x) = 1 - \sin^2 x$

$4\sin^3 x + \sin^2 x - 3\sin x = 0$

$\sin x(4\sin^2 x + \sin x - 3) = 0$

$\sin x(4\sin x - 3)(\sin x + 1) = 0$

$\sin x = 0$ or	$\sin x = {}^{3}/_{4}$ or	$\sin x = (-1)$
$x = -180, 0, 180, 360$	$x = 48.6, 131.4$	$x = -90, 270$

16.

(a) Solve and find $V(t)$

Integrate:

$$\frac{dV}{dt} = 10te^{-t} \Rightarrow V = \int 10te^{-t}\,dt$$

Use integration by parts on $\int te^{-t}dt$:

Let $u = t \Rightarrow du = dt$

Let $dv = e^{-t}dt \Rightarrow v = -e^{-t}$

$$\int te^{-t}dt = t(-e^{-t}) - \int 1(-e^{-t})dt = -te^{-t} + \int e^{-t}dt = -te^{-t} - e^{-t} + C$$

$$V = 10(-te^{-t} - e^{-t}) + C = -10te^{-t} - 10e^{-t} + C$$

Initial condition: $V(0) = 4$

At $t = 0$, $e^0 = 1$:

$$4 = -10(0)(1) - 10(1) + C = -10 + C \Rightarrow C = 14$$

$$\boxed{V(t) = 14 - 10e^{-t}(t+1)}$$

(b) Will the tank ever become full (15 L)?

As $t \to \infty$, $e^{-t} \to 0$, and $te^{-t} \to 0$.

So:

$$V(t) \to 14$$

The maximum volume according to the model is 14 litres, which is less than the capacity 15 litres.

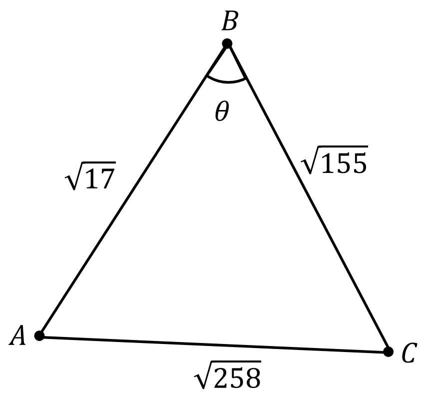
$$\boxed{\text{No, the tank will never become full, because } V(t) \to 14 < 15.}$$

17.

a) $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$

$= 2i + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

b)



$$|\overrightarrow{AB}| = \sqrt{4+9+4}$$
$$= \sqrt{17}$$

$\overrightarrow{BC} = 3i + 5\underset{\sim}{j} + 11\underset{\sim}{k}$

$$|\overrightarrow{BC}| = \sqrt{9+25+121}$$
$$= \sqrt{155}$$

$\overrightarrow{AC} = 5i + 8\underset{\sim}{j} + 13\underset{\sim}{k}$

$|\overrightarrow{AC}| = \sqrt{25 + 64 + 169}$

$= \sqrt{258}$

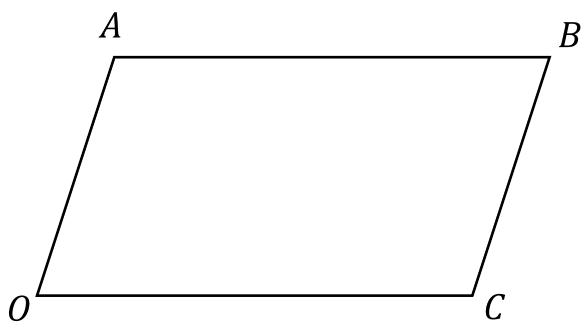
$$\cos\theta = \frac{(\sqrt{17})^2 + (\sqrt{155})^2 - (\sqrt{258})^2}{2\sqrt{17} \times \sqrt{155}}$$

$$= \frac{17 + 155 - 258}{2\sqrt{17} \times \sqrt{155}}$$

$\cos\theta = -0.8376$

$\theta = 146.9°$

c)



$\overrightarrow{OC} = 8i + 12\underset{\sim}{j} + 8\underset{\sim}{k}$

$\overrightarrow{AB} = 2i + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

$\overrightarrow{OC} = 4\overrightarrow{AB}$

$OABC$ is a trapezium

END

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2026 (June)
Statistics and Mechanics (Set 1)

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This AL Maths paper 3 (Statistics and Mechanics: Predicted Paper June 2026: Set 1) has been created based on the most common topics from previous past papers. This paper should be excellent for helping students revise for exams; however, it should not be relied upon as the sole basis for revision

Grade Boundary: A: 61% and A*: 77%

1.

A researcher is investigating the relationship between average income x(GDP per capita, US dollars) and average annual CO_2 emissions y(tonnes) for different countries.

A random sample of 20 countries gives a product moment correlation coefficient of

$$r = 0.41$$

(a) Stating your hypotheses clearly, test at the 5% level of significance whether the correlation coefficient for all countries is greater than 0.

The researcher thinks a non-linear model may fit better. She codes the data using

$$m = \log_{10} x, c = \log_{10} y$$

and obtains the model

$$c = -2.05 + 0.76m$$

The correlation coefficient between c and m is found to be

$$r = 0.90$$

(b) Explain how this value supports the researcher's belief

(c) Show that the relationship between $y$and $x$can be written in the form

$$y = ax^n$$

where a and n are constants to be found.

(7)

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2.

In a university, 6% of students are members of the university chess club. A random sample of 45 students is taken.

(a) Define a suitable probability distribution model for the number of students in the sample who are members of the chess club.

(b) Using this model: Calculate $P(X = 3)$, where X represents the number of students in the sample who are members of the chess club. And calculate $P(X \geq 5)$

(c) Only 35% of the chess club members have won a chess tournament. Find the probability that a randomly selected student is a chess club member who has won a tournament.

(d) In a sample of 60 students, find the probability that no more than 2 students are chess club members who have won a tournament.

(10)

3.

Heights of females in a country are normally distributed with

- mean $\mu = 168.0$cm
- standard deviation $\sigma = 6.5$cm

Given that 2% of females are shorter than $k$cm,

(a) find k.

(b) Find the proportion of females with heights between 155 cm and 180 cm.

A female is chosen at random from those with heights between 155 cm and 180 cm.

(c) Find the probability that her height is more than 165 cm.

Heights of females in a different country are normally distributed with standard deviation 7.8cm.

Lia believes the mean height in this country is less than 168.0 cm.

A random sample of 60 females has sample mean $\bar{x} = 166.9$cm.

(d) Carry out a suitable test of Lia's belief at the 5% significance level.

(11)

4.

The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{64} a$	$\log_{64} b$	$\log_{64} c$

where

- a, b, c are distinct integers with $a < b < c$
- all the probabilities are greater than 0

(a) Find

(i) a

(ii) b

(iii) c

Show your working clearly.

Independent random variables X_1 and X_2 each have the same distribution as X.

(b) Find $P(X_1 = X_2)$.

(6)

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5.

Mina is studying the large data set for April 2016.

She codes the variable Daily Mean Pressure, x, using

$$y = x - 1010$$

The data for all 30 days from a location are summarised by

$$\sum y = -60, \sum y^2 = 7200$$

(a) State the units of the variable x.

(b) Find the mean Daily Mean Pressure for these 30 days.

(c) Find the standard deviation of Daily Mean Pressure for these 30 days.

Mina knows that, in the UK, winds circulate

- clockwise around a region of high pressure
- anticlockwise around a region of low pressure

On 18/04/2016 the Daily Mean Pressure for 3 locations was:

Location	Heathrow	Hurn	Leuchars
Daily Mean Pressure	1030	1026	1028
Cardinal Wind Direction			

The Cardinal Wind Directions for these 3 locations (in random order) were:

W, E, NE

You may assume that these 3 locations were under a single region of pressure.

(d) Place each wind direction into the correct location in the table and give a reason.

(8)

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6.

A library knows from past records that 12% of users return books late.

A new librarian believes that the proportion of users who return books late has changed.

A random sample of 40 library users is taken.

Let X be the number of users in the sample who return books late.

(a) Write down

- a null hypothesis corresponding to no change in the proportion of late returns,
- an alternative hypothesis corresponding to the librarian's belief.

(b) Using a 5% level of significance, find a critical region for a two-tailed test of the null hypothesis.

State the probability in each tail (each should be less than 0.025).

(c) Find the actual level of significance of the test based on your critical region.

The librarian observes that 8 of the 40 users return books late.

(d) With reference to part (b), comment on the librarian's belief.

(8)

7.

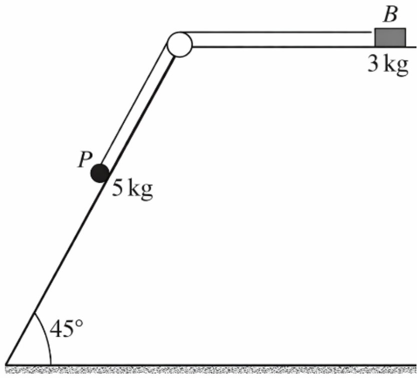
A cyclist starts from rest and accelerates uniformly at 2.5 m s^{-2} along a straight road.

(a) Find the speed of the cyclist after 6s.

(b) Find the distance travelled in the first 6s.

(4)

8.



A small block B of mass 3 kg and a particle P of mass 5 kg are attached to the ends of a light inextensible string. The string passes over a small smooth pulley fixed at the intersection of a rough horizontal surface and a rough plane inclined at 45°to the horizontal.

The system is released from rest. In the subsequent motion, P moves down the plane and B moves towards the pulley (but does not reach it).

The coefficient of friction between P and the plane is 0.3 and the coefficient of friction between B and the horizontal surface is 0.4. Take $g = 9.8\text{ m s}^{-2}$.

(a) Find the tension Tin the string (in newtons)

When $P$is moving at 10 m s^{-1}, the string breaks.

(b) Find the deceleration of B after the string breaks.

(12)

9.

(i) At time $t$seconds ($t \geq 0$), a particle P has acceleration

$$\mathrm{a} = (2 - 6t)\mathrm{i} + (4 - t^2)\mathrm{j}\ \mathrm{ms}^{-2}$$

At $t = 0$, the velocity of P is $5\mathrm{i} - 2\mathrm{j}\ \mathrm{ms}^{-1}$

(a) Find v when $t = 3$.

(b) Find the value of $t$when $P$is moving in a direction perpendicular to i.

(ii) At time $t$seconds ($t \geq 0$), a particle Q has position vector

$$\mathrm{r} = (t^2 + 2t)\mathrm{i} + (4t)\mathrm{j}\ \mathrm{m}.$$

Find the value(s) of t when the speed of Q is $10\ \mathrm{m\ s}^{-1}$.

(9)

10.

A particle of mass 5 kg is at rest on a smooth horizontal plane at point A. Two constant forces

$$F_1 = (6i + 2j)\ \text{N}, F_2 = (\lambda i + \mu j)\ \text{N}$$

act on it.

The particle moves in the direction of the vector $(2i - j)$.

(a) Show that $\lambda + 2\mu + 10 = 0$.
(b) Given $\lambda = 0$, find the distance AB travelled in $t = 3$ seconds.

(8)

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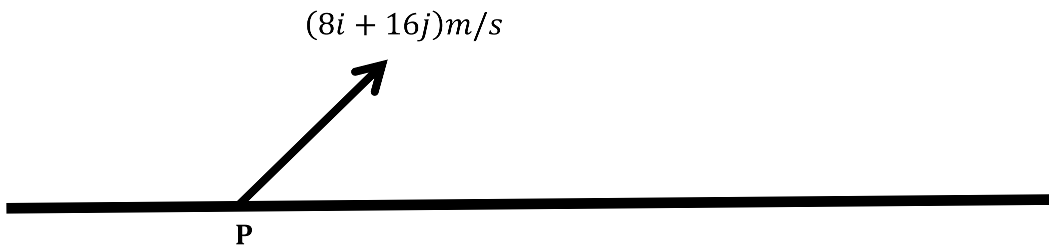
11.



A particle is projected with an initial velocity $(8i + 16j)$ m/s from a point on level ground. Take $g = 10 \text{ m s}^{-2}$

(a) Show that the trajectory of the particle can be expressed as:

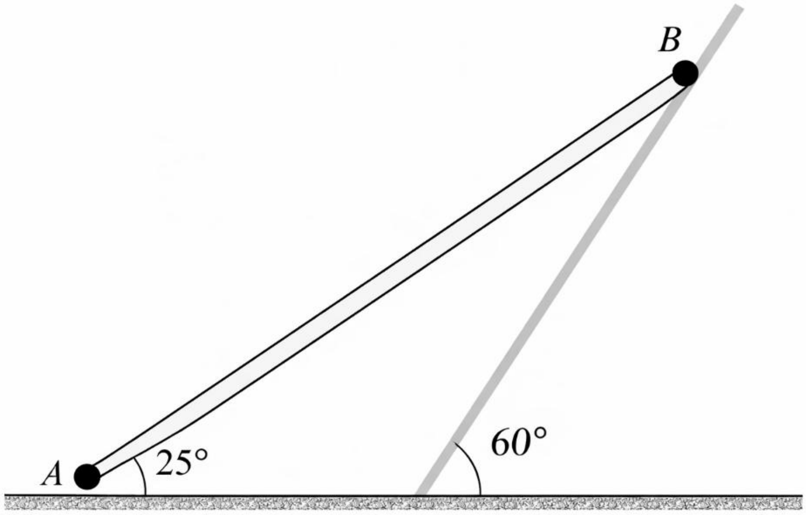
$y = \frac{1}{64}x(128 - 5x)$

(b) Calculate the particle's range, maximum height, and the time of flight.

(c) A point Q lies on the trajectory such that the velocity vector at Q is perpendicular to the initial projection velocity. Find the coordinates of Q.

(7)

12.



A uniform rod AB of weight 24 N and length 3.0 m rests in equilibrium with end A in contact with rough horizontal ground and end B resting against a smooth wall inclined at 60°to the horizontal.

The rod lies in a vertical plane perpendicular to the wall and is inclined at 25°to the horizontal.

(a) Show that the magnitude of the force acting on the rod at $B$is 13.3 N
(b) Determine the magnitude of the contact force between the rod and the ground at A.

(10)

END

TOTAL FOR PAPER IS 100 MARKS

A Level Maths Predicted Paper 2026 (June) Statistics and Mechanics: Paper 3: Set 1: Answers

1.

(a) Hypothesis test for positive correlation

Let ρbe the population (true) correlation coefficient.

$$H_0: \rho = 0 \; H_1: \rho > 0$$

Critical value:

$$0.378$$

Since

$$0.41 > 0.378$$

reject H_0. There is evidence that the correlation is greater than 0.

Conclusion: There is sufficient evidence at the 5% level that $\rho > 0$: a positive correlation exists.

(b) Why $r = 0.90$ supports the non-linear model

After taking logs, the correlation becomes much stronger (from about 0.41 to 0.90), meaning the log–log transformation makes the relationship closer to a straight line. This suggests the original x–y relationship is non-linear (specifically power-type).

(c) Show $y = ax^n$and find a, n

Given:

$$c = \log_{10} y, m = \log_{10} x$$

and the fitted line:

$$c = -2.05 + 0.76m$$

Substitute back:

$$\log_{10} y = -2.05 + 0.76\log_{10} x$$

Rewrite using log rules:

$$\log_{10} y = \log_{10}(10^{-2.05}) + \log_{10}(x^{0.76})$$
$$\log_{10} y = \log_{10}(10^{-2.05}x^{0.76})$$
$$y = 10^{-2.05}x^{0.76}$$

Hence it is of the form $y = ax^n$with

$$\boxed{a = 10^{-2.05}}, \boxed{n = 0.76}$$

2.

(a) Distribution model

Each student is (approximately) independent, with probability $p = 0.06$of being a chess club member. For $n = 45$:

$$X \sim \text{Bin}(45, 0.06)$$

(b) Calculate $P(X = 3)$and $P(X \geq 5)$

$$P(X = 3)$$

$$P(X = 3) = \binom{45}{3}(0.06)^3(0.94)^{42} \approx 0.220$$

$$P(X \geq 5)$$

$$P(X \geq 5) = 1 - P(X \leq 4)$$

$$= 1 - \sum_{k=0}^{4} \binom{45}{k} (0.06)^k (0.94)^{45-k} \approx 1 - 0.9415 \approx 0.0585$$

$$\boxed{P(X = 3) \approx 0.220, P(X \geq 5) \approx 0.0585}$$

(c) Probability a student is a member and has won a tournament

$$P(\text{member and winner}) = P(\text{member}) \cdot P(\text{winner} \mid \text{member}) = 0.06 \times 0.35 = 0.021$$

\boxed{0.021\text{ (2.1%)}}

(d) In a sample of 60, probability no more than 2 are members who have won

Let Y be the number of "member-and-winner" students in a sample of 60.

From (c), probability a student is a member-and-winner is

$$p = 0.021$$

$$Y \sim \text{Bin}(60, 0.021)$$

$$P(Y \leq 2) = \sum_{k=0}^{2} \binom{60}{k} (0.021)^k (0.979)^{60-k} \approx 0.956$$

$$\boxed{P(Y \leq 2) \approx 0.956}$$

3.

Answers

Let $X \sim N(168, 6.5^2)$.

(a) Find k given $P(X < k) = 0.02$

$$P\left(Z < \frac{k - 168}{6.5}\right) = 0.02 \Rightarrow \frac{k - 168}{6.5} = z_{0.02} \approx -2.054$$

$$k = 168 + (-2.054)(6.5) \approx 168 - 13.35 = 154.65$$

$$\boxed{k \approx 154.7 \text{ cm}}$$

(b) Proportion between 155 cm and 180 cm

$$P(155 < X < 180) = \Phi\left(\frac{180 - 168}{6.5}\right) - \Phi\left(\frac{155 - 168}{6.5}\right)$$

$$= \Phi(1.846) - \Phi(-2.000) \approx 0.9676 - 0.0228 = 0.9448$$

$$\boxed{0.945 \text{ (approximately)}}$$

(c) Conditional probability $P(X > 165 \mid 155 < X < 180)$

$$P(X > 165 \mid 155 < X < 180) = \frac{P(165 < X < 180)}{P(155 < X < 180)}$$

Denominator from (b): ≈ 0.9448

Numerator:

$$P(165 < X < 180) = \Phi(1.846) - \Phi\left(\frac{165 - 168}{6.5}\right)$$

$$\frac{165 - 168}{6.5} = -0.462, \Phi(-0.462) \approx 0.3222$$

$$\Rightarrow 0.9676 - 0.3222 = 0.6454$$

$$\text{So } \frac{0.6454}{0.9448} \approx 0.683$$

$$\boxed{0.683 \text{ (approximately)}}$$

(d) Hypothesis test (5% level)

Given $\sigma = 7.8$, $n = 60$, $\bar{x} = 166.9$, test mean < 168.

Hypotheses

$$H_0: \mu = 168 \; H_1: \mu < 168$$

Test statistic (z-test, σknown):

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{166.9 - 168}{7.8/\sqrt{60}}$$

$$7.8/\sqrt{60} \approx 7.8/7.746 \approx 1.007$$

$$z \approx \frac{-1.1}{1.007} \approx -1.09$$

Critical value for 5% one-tailed: -1.645.

Since $-1.09 > -1.645$, we do not reject H_0.

(Equivalently, p-value $\approx \Phi(-1.09) \approx 0.14 > 0.05$.)

4.

(a)

Because probabilities sum to 1:

$$\log_{64} a + \log_{64} b + \log_{64} c = 1$$

Use log laws:

$$\log_{64}(abc) = 1 \;\; \Rightarrow \;\; abc = 64$$

Also, each probability is > 0, so each log is > 0, meaning:

$$a > 1,\; b > 1,\; c > 1$$

Now factor 64:

$$64 = 2^6$$

A suitable set of distinct integers $a < b < c$with product 64is:

$$a = 2, b = 4, c = 8$$

(Check: $2 \cdot 4 \cdot 8 = 64$.)

$$\boxed{a = 2,\; b = 4,\; c = 8}$$

(Then the probabilities are $\log_{64} 2 = \frac{1}{6}$, $\log_{64} 4 = \frac{1}{3}$, $\log_{64} 8 = \frac{1}{2}$, which sum to 1.)

(b) $P(X_1 = X_2)$

$$P(X_1 = X_2) = \sum P(X = x)^2$$

$$= (\frac{1}{6})^2 + (\frac{1}{3})^2 + (\frac{1}{2})^2 = \frac{1}{36} + \frac{1}{9} + \frac{1}{4}$$

$$= \frac{1}{36} + \frac{4}{36} + \frac{9}{36} = \frac{14}{36} = \frac{7}{18}$$

$$\boxed{P(X_1 = X_2) = \frac{7}{18}}$$

5.

(a)

Units: hectopascals (hPa) (equivalently millibars, mb).

(b) Mean Daily Mean Pressure

$$\bar{y} = \frac{\sum y}{n} = \frac{-60}{30} = -2$$

Since $y = x - 1010$, then $x = y + 1010$, so

$$\bar{x} = \bar{y} + 1010 = -2 + 1010 = 1008$$

Mean Daily Mean Pressure = $\boxed{1008 \text{ hPa}}$

(c) Standard deviation

Use

$$s = \sqrt{\frac{\sum y^2}{n} - \bar{y}^2}$$

$$s = \sqrt{\frac{7200}{30} - (-2)^2} = \sqrt{240 - 4} = \sqrt{236} \approx 15.36$$

Standard deviation of $x$is the same as standard deviation of y(subtracting 1010 shifts data but doesn't change spread).

$$\boxed{s \approx 15.4 \text{ hPa}}$$

(d) Wind directions in the table

Since Heathrow has the highest pressure (1030 hPa), we assume the centre of a high is closest to Heathrow. Winds around a high pressure system in the UK are clockwise.

With Heathrow roughly between Leuchars (north) and Hurn (south):

- North of a high → wind tends to be from the East (E)
- South of a high → wind tends to be from the West (W)

Location	Heathrow	Hurn	Leuchars
Cardinal Wind Direction	NE	W	E

Reason (example wording): Heathrow has the highest pressure so is nearest the high-pressure centre; winds circulate clockwise around a high, giving E to the north (Leuchars) and W to the south (Hurn), leaving NE for Heathrow.

6.

(a) Hypotheses

Let $p$be the true proportion of users who return books late.

$$\boxed{H_0: p = 0.12}$$

$$\boxed{H_1: p \neq 0.12}$$

(b) Critical region (5% two-tailed test)

Under H_0,

$$X \sim \text{Bin}(40, 0.12).$$

We require each tail probability to be less than 0.025.

Lower tail

$$P(X \leq 1) = P(X = 0) + P(X = 1) \approx 0.0146 < 0.025$$

but

$$P(X \leq 2) \approx 0.0615 > 0.025$$

So the lower critical region is:

$$\boxed{X \leq 1}$$

Upper tail

$$P(X \geq 9) \approx 0.0218 < 0.025$$

but

$$P(X \geq 8) \approx 0.0447 > 0.025$$

So the upper critical region is:

$$\boxed{X \geq 9}$$

Critical region

$$\boxed{X \leq 1 \text{ or } X \geq 9}$$

(c) Actual level of significance

$$\alpha = P(X \leq 1) + P(X \geq 9)$$
$$\alpha \approx 0.0146 + 0.0218 = 0.0364$$
$$\boxed{\text{Actual significance } \approx 0.0364}$$

(d) Comment on the librarian's belief

Observed value:

$$X = 8$$

Since

$$1 < 8 < 9,$$

the observed value does not lie in the critical region.

Conclusion

We do not reject H_0.

There is insufficient evidence at the 5% level to conclude that the proportion of users who return books late has changed.

$$\boxed{\text{The data do not support the librarian's belief.}}$$

7.

$$u = 0, a = 2.5, t = 6$$

(a)

$$v = u + at = 0 + 2.5(6) = 15$$
$$\boxed{v = 15 \text{ m s}^{-1}}$$

(b)

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}(2.5)(6^2) = 1.25(36) = 45$$
$$\boxed{s = 45 \text{ m}}$$

8.

- $m_B = 3$ kg, on rough horizontal, $\mu_B = 0.4$
- $m_P = 5$ kg, on rough plane at 45°, $\mu_P = 0.3$
- $g = 9.8 \text{ m s}^{-2}$
- While string is intact: Pmoves down the plane, Bmoves towards the pulley.

(a) Tension T in the string

Let the common acceleration be a(down the plane for P, towards pulley for B).

For P(down the plane positive)

Normal reaction:

$$R_P = 5g\cos\ 45^\circ$$

Friction (up the plane):

$$F_P = \mu_P R_P = 0.3(5g\cos\ 45^\circ)$$

Equation along plane:

$$5a = 5g\sin\ 45^\circ - T - 0.3(5g\cos\ 45^\circ)$$

Since $\sin\ 45^\circ = \cos\ 45^\circ = \frac{1}{\sqrt{2}}$:

$$5a = \frac{5g}{\sqrt{2}} - T - \frac{0.3 \cdot 5g}{\sqrt{2}} = \frac{5g(1-0.3)}{\sqrt{2}} - T = \frac{3.5g}{\sqrt{2}} - T$$

$$5a = \frac{3.5g}{\sqrt{2}} - T(1)$$

For B(towards pulley positive)

Normal reaction $= 3g$. Friction opposing motion:

$$F_B = \mu_B(3g) = 0.4(3g) = 1.2g$$

Equation horizontally:

$$3a = T - 1.2g(2)$$

$$T = 3a + 1.2g$$

$$5a = \frac{3.5g}{\sqrt{2}} - (3a + 1.2g)$$

$$a = 1.5625$$

$$T = 3a + 1.2g$$

$$\boxed{T \approx 16.4\ \text{N}}$$

(b) Deceleration of Bafter the string breaks

After the string breaks, $B$is no longer pulled by tension, so the only horizontal force is friction:

$$F_B = \mu_B(3g) = 1.2g$$

This acts opposite to the motion, so the deceleration magnitude is

$$a_B = \frac{F_B}{m_B} = \frac{1.2g}{3} = 0.4g$$

$$a_B = 0.4(9.8) = 3.92ms^{-2}$$

9.

(i)(a) Velocity at $t = 3$

$$\text{v}'(t) = \text{a}(t)$$

x-component

$$v_x = \int (2 - 6t)\,dt = 2t - 3t^2 + C_1$$

At $t = 0$, $v_x(0) = 5 \Rightarrow C_1 = 5$.

$$v_x = 2t - 3t^2 + 5$$

y-component

$$v_y = \int (4 - t^2)\, dt = 4t - \frac{t^3}{3} + C_2$$

At $t = 0$, $v_y(0) = -2 \Rightarrow C_2 = -2$.

$$v_y = 4t - \frac{t^3}{3} - 2$$

$$\mathrm{v}(t) = (2t - 3t^2 + 5)\mathrm{i} + (4t - \frac{t^3}{3} - 2)\mathrm{j}$$

At $t = 3$:

$$v_x(3) = 6 - 27 + 5 = -16$$
$$v_y(3) = 12 - \frac{27}{3} - 2 = 12 - 9 - 2 = 1$$
$$\mathrm{v}(3) = -16\mathrm{i} + \mathrm{j}\ \mathrm{ms}^{-1}$$

(i)(b) Perpendicular to i

"Perpendicular to i" means velocity has no i-component, so $v_x = 0$:

$$2t - 3t^2 + 5 = 0$$
$$-3t^2 + 2t + 5 = 0 \ \Rightarrow \ 3t^2 - 2t - 5 = 0$$
$$t = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-5)}}{2 \cdot 3} = \frac{2 \pm \sqrt{4 + 60}}{6} = \frac{2 \pm 8}{6}$$

So $t = \frac{10}{6} = \frac{5}{3}$or $t = -1$. Since $t \geq 0$:

$$\boxed{t = \frac{5}{3}\ \mathrm{s}}$$

(ii) Speed of Q

$$\mathrm{v} = \frac{d\mathrm{r}}{dt} = (2t + 2)\mathrm{i} + 4\mathrm{j}$$

Speed:

$$|\ \mathrm{v}\ |= \sqrt{(2t + 2)^2 + 4^2} = 10$$

Square both sides:

$$(2t + 2)^2 + 16 = 100$$
$$(2t + 2)^2 = 84$$
$$2t + 2 = \pm\sqrt{84} = \pm 2\sqrt{21}$$
$$\boxed{t = -1 + \sqrt{21}\ \mathrm{s}}$$

10.

Resultant force

$$\mathrm{F} = \mathrm{F}_1 + \mathrm{F}_2 = ((6 + \lambda)\mathrm{i} + (2 + \mu)\mathrm{j})$$

Acceleration is in direction F, and motion direction is $(2, -1)$, so components are proportional:

$$\frac{2 + \mu}{6 + \lambda} = \frac{-1}{2}$$
$$2(2 + \mu) = -(6 + \lambda) \Rightarrow 4 + 2\mu = -6 - \lambda \Rightarrow \boxed{\lambda + 2\mu + 10 = 0}$$

For (b), $\lambda = 0 \Rightarrow 2\mu + 10 = 0 \Rightarrow \mu = -5$.

$$F = (6i + 2j) + (0i - 5j) = 6i - 3j$$

$$a = \frac{F}{m} = \frac{1}{5}(6i - 3j) = (\frac{6}{5}i - \frac{3}{5}j)$$

Starting from rest, displacement after time t:

$$s = \frac{1}{2}at^2$$

With $t = 3$:

$$s = \frac{1}{2}(\frac{6}{5}i - \frac{3}{5}j)9 = (\frac{27}{5}i - \frac{27}{10}j)$$

$$AB = | s | = \sqrt{\left(\frac{27}{5}\right)^2 + \left(\frac{27}{10}\right)^2} = 27\sqrt{\frac{1}{25} + \frac{1}{100}} = 27\sqrt{\frac{5}{100}} = \frac{27\sqrt{5}}{10}\ \text{m}$$

11.

a) Show the equation of the trajectory

Horizontal and vertical motion:

- Horizontal: $x = 8t$
- Vertical: $y = 16t - \frac{1}{2}(10)t^2 = 16t - 5t^2$

Eliminate $t$using $t = \frac{x}{8}$:

$$y = 16(\frac{x}{8}) - 5(\frac{x}{8})^2 = 2x - 5 \cdot \frac{x^2}{64} = 2x - \frac{5x^2}{64}$$

Factor:

$$y = \frac{1}{64}(128x - 5x^2) = \frac{1}{64}x(128 - 5x)$$

$$\boxed{y = \frac{1}{64}x(128 - 5x)}$$

(b) Range, maximum height, time of flight

Range

Range is where $y = 0$ (other than $x = 0$):

$$\frac{1}{64}x(128 - 5x) = 0 \Rightarrow x = 0 \text{ or } x = 25.6$$

$$\boxed{\text{Range } = 25.8\ \text{m}}$$

Maximum height

The parabola $y = \frac{1}{64}x(128 - 5x)$is symmetric about $x = 12.8$, so maximum at $x = 12.8$:

$$y_{\max} = \frac{1}{64} * 12.8 * (128 - 5 * 128) = 12.8\text{m}$$

$$\boxed{\text{Maximum height } = 12.8\ \text{m}}$$

Time of flight

At landing, $x = 25.6m$ and $x = 8t$:

$$25.6 = 8t \Rightarrow t = 3.2$$

$$\boxed{\text{Time of flight } = 3.2\ \text{s}}$$

(c)
Given
Initial velocity: (8i + 16j) m s^{-1}
So the initial direction gradient is

$$m_0 = \frac{u_y}{u_x} = \frac{16}{8} = 2.$$

For a projectile,

$$v_x = 8, v_y = 16 - 8t$$

so the gradient of the trajectory (direction of velocity) at time $t$is

$$\frac{dy}{dx} = \frac{v_y}{v_x} = \frac{16 - 8t}{8} = 2 - t.$$
$$m_1 m_2 = -1.$$
$$2 - t = -\frac{1}{2}$$
$$t = 2 + \frac{1}{2} = \frac{5}{2} = 2.5.$$

$$x = 8t = 8(2.5) = 20.$$
$$y = 16t - 4t^2 = 16(2.5) - 4(2.5)^2 = 40 - 25 = 15.$$
Q(20, 15)

12.
(a) Show $R_B = 13.3$ N
Take moments about A.

- Moment of Wabout A: perpendicular distance is the horizontal distance of the midpoint:

$$(\frac{L}{2})\cos 25^\circ = 1.5\cos 25^\circ$$

Clockwise moment:

$$24(1.5\cos 25^\circ)$$

Anticlockwise moment:

$$R_B(3\sin 55^\circ)$$

Equilibrium:

$$R_B(3\sin 55^\circ) = 24(1.5\cos 25^\circ)$$

$$R_B = \frac{12\cos 25^\circ}{\sin 55^\circ}$$
$$13.28 \approx \boxed{13.3 \text{ N}}$$

(b) Contact force between rod and ground at A
Resolve forces.
Components of R_B

$$R_{Bx} = R_B\cos 30^\circ = 13.28(0.8660) = 11.50$$
$$R_{By} = R_B\sin 30^\circ = 13.28(0.5) = 6.64$$

Horizontal equilibrium

$$F = R_{Bx} \Rightarrow F = 11.50 \text{ N}$$

Vertical equilibrium

$$N + R_{By} = 24 \Rightarrow N = 24 - 6.64 = 17.36 \text{ N}$$

Resultant contact force at A

$$R_A = \sqrt{N^2 + F^2} = \sqrt{(17.36)^2 + (11.50)^2}$$

$$R_A = \sqrt{301.4 + 132.3} = \sqrt{433.7} = 20.82 \approx \boxed{20.8 \text{ N}}$$

END